C12 - 9.0 - Rationals Review

$$\textbf{Rational} = \frac{Polynomial}{Polynomial}$$

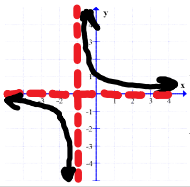
Holes: Factor the top, Factor the bottom. If a factor cancels, there is a hole when the factor equals zero.

$y = \dfrac{\cancel{x-3}}{\cancel{(x-3)}(x+2)} = \dfrac{1}{x+2}$ $\quad x-3=0,\ x=3$ $\quad y = \dfrac{1}{(3)+2} = \dfrac{1}{5}$ $\quad Hole: \left(3, \dfrac{1}{5}\right)$

$Domain: x \neq 3$
$Range: Depends$

Vertical Asymptote('s): $denominator = 0$

$y = \dfrac{1}{x+1}$ $\quad x+1=0,\ x=-1$



Set denominator equal to zero and solve.

$VA: x=-1$
$D: x \neq -1$

Domain: $x \neq VA, Holes$

TOV

	x	y
VA	-2	-1
	-1	und
	0	1

Behavior near Asymptote

$x \to -1^+,\ y \to \infty$
$x \to -1^-,\ y \to -\infty$

Point('s) on Both sides of VA('s)

Horizontal Asymptote:

Case 1: Higher on Top

$x^2, \dfrac{x^2}{x}$

$HA:$ *none*

Case 2: Higher on Bottom

$\dfrac{1}{x}, \dfrac{1}{x^2}$ $\quad HA: y=0$

$\dfrac{1}{x}+c, \dfrac{1}{x^2}+c$ $\quad HA: y=c$

Case 3: Same Degree

$\dfrac{3x^2}{2x^2}$ $\quad HA: y=\dfrac{3}{2}$

$\dfrac{3x^2}{2x^2}+c$ $\quad HA: y=\dfrac{3}{2}+c$

End Behavior

x	y
$-\infty$	$?$
∞	$?$

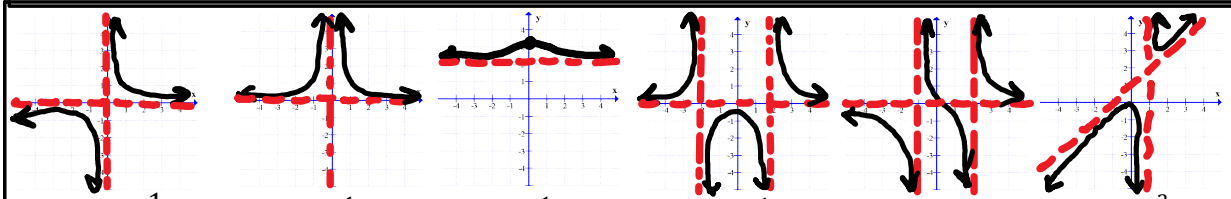
$x \to \infty,\ y \to ?^+$
$x \to -\infty,\ y \to ?^-$

Graph Steps*:
$Holes, VA, HA, SA$
$TOV, x-int, y-int$
Close to asymptote
Through point('s)
Close to asymptote

Range Restrictions: $Depends$
$y \neq HA^*, Holes^*$

Slant Asymptote: Case#1
Long/Synthetic Division
If: Linear Quotient = Slant Asymptote
Eg. $mx+b$

Common Rational Graphs



$y = \dfrac{1}{x}$ $\quad y = \dfrac{1}{x^2}$ $\quad y = \dfrac{1}{x^2+1}+2$ $\quad y = \dfrac{1}{x^2-4}$ $\quad y = \dfrac{x}{x^2-2x-3}$ $\quad y = \dfrac{x^2}{x-1}$

$\dfrac{2}{x-1}+3$

Add Fractions

$\dfrac{2}{x-1}+3$

$\dfrac{2}{x-1}+3\times\dfrac{x-1}{x-1}$

$\dfrac{2}{x-1}+\dfrac{3x-3}{x-1}$

$\dfrac{3x-1}{x-1}$

Long Division

$$x-1\ \overline{)\ 3x-1}$$
$$\underline{3x-3}$$
$$2$$

$3+\dfrac{2}{x-1}$

Synthetic Division

$x-1=0$
$x=+1$

$3x-1$

$+1$ | 3 $\quad -1$
$\quad\ \downarrow$ $\quad +3$
3 $\quad 2$

$3\ \ R:2$

They are the Same!

$\dfrac{2}{x-1}+3=\dfrac{3x-1}{x-1}$

$$y = \frac{a}{x-h}+k$$

$$y = \frac{a}{VA}+HA$$

$$y = \frac{a(HA)(x-int)(holes)}{(HA)(VA's)(holes)}$$