

$$F_g = \frac{Gm_1m_2}{r^2}$$

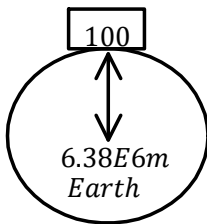
$$E_{p\infty} = -\frac{GMm}{r}$$

P12 - 7.1 - Fg Ep Notes

F_g : The Gravitational Force between any two Objects anywhere in the Universe. (Newton)

E_p : The Potential Energy a massive object has in relation to another due to gravity. (Joules)

Distance from Earth



$$F_g = \frac{Gm_1m_2}{r^2}$$

$$g = \frac{GM}{r^2}$$

$$F_g = \frac{6.67 \times 10^{-11}(5.98 \times 10^{24})(100)}{(6.38 \times 10^6)^2}$$

$$F_g = 979.9 \text{ N}$$

$$F_g = mg$$

$$F_g = 100 \times 9.8$$

$$F_g = 980 \text{ N}$$

$$E_p = gmr$$

$$W = mgh$$

Obviously:)

$$E_p = -\frac{GMm}{r}$$

$$E_p = -\frac{6.67 \times 10^{-11}(5.98 \times 10^{24})(100)}{6.38 \times 10^6}$$

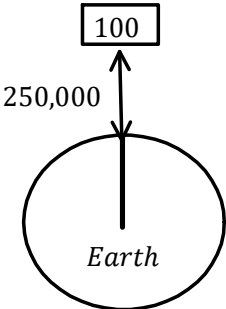
$$E_p = -6.25 \times 10^9 \text{ J}$$

$$E_p = mgh$$

$$E_p = mg(0)$$

$$E_p = 0 \text{ J}$$

$$mgh = E_{pf} - E_{pi}; r < 10000m$$



$$F_g = \frac{Gm_1m_2}{r^2}$$

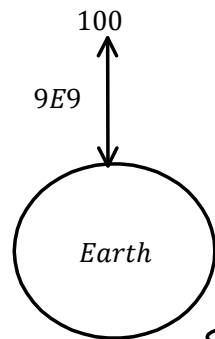
$$F_g = \frac{6.67E-11(5.98E24)(100)}{(6.38E6 + 250000)^2}$$

$$F_g = 907.4 \text{ N}$$

$$E_p = -\frac{GMm}{r}$$

$$E_p = -\frac{6.67E-11(5.98E24)(100)}{6.38E6 + 250000}$$

$$E_p = -6.01E9 \text{ J}$$



$$F_g = \frac{Gm_1m_2}{r^2}$$

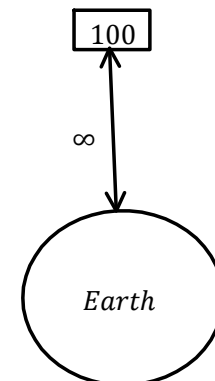
$$F_g = \frac{6.67E-11(5.98E24)(100)}{(6.38E6 + 9E9)^2}$$

$$F_g = 4.92E-4 \text{ N}$$

$$E_p = -\frac{GMm}{r}$$

$$E_p = -\frac{6.67E-11(5.98E24)(100)}{6.38E6 + 9E9}$$

$$E_p = -2.58E6 \text{ J}$$



$$F_g = \frac{Gm_1m_2}{r^2}$$

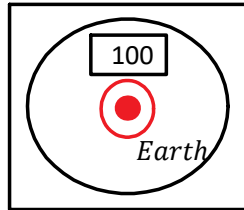
$$F_g = \frac{6.67E-11(5.98E24)(100)}{(6.38E6 + \infty)^2}$$

$$F_g = 0 \text{ N}$$

$$E_{p\infty} = -\frac{GMm}{r}$$

$$E_{p\infty} = -\frac{6.67E-11(5.98E24)(100)}{6.38E6 + \infty}$$

$$E_{p\infty} = 0 \text{ J}$$



Near Centre

$$F_g \approx \infty \text{ N}$$

Surface

$$F_g = 980 \text{ N}$$

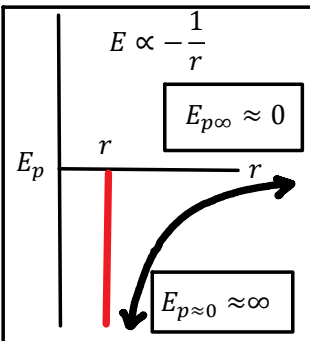
Far Away

$$F_g \approx 0.0005 \text{ N}$$

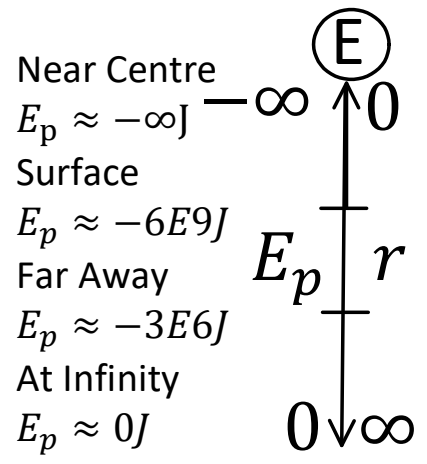
At Infinity

$$F_g \approx 0 \text{ N}$$

Weaker



E_p : Farther from Earth less negative!



P12 - 7.2 - Fg Ep Work to Height Notes

What is the Gravitational Force, F_g , and Potential Energy between twins of 50kg 5 m apart?

$$F_g = \frac{Gm_1m_2}{r^2}$$

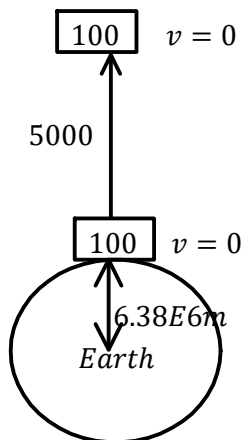
$$F_g = \frac{(6.67 \times 10^{-11})(50)(50)}{(5)^2}$$

$$F_g = 6.67 \times 10^{-9} \text{ N}$$

$$E_p = -\frac{GMm}{r}$$

$$E_p = -\frac{6.67E-11(50)(50)}{5}$$

$$E_p = 3.36E-8 \text{ J}$$



$$W = \Delta E$$

$$W = \Delta E_k + \Delta E_p$$

$$W = \Delta E_p \quad ; E_k = 0$$

$$W = E_{pf} - E_{pi}$$

$$W = -\frac{GMm}{r_f} - \left(-\frac{GMm}{r_i}\right)$$

$$W = -\frac{6.67E-11(5.98E24)(100)}{6.38E6 + 5000} - \left(-\frac{6.67E-11(5.98E24)(100)}{6.38E6}\right)$$

$$W = -6246922475 - -6251818182 \text{ J}$$

$$W = 4895707$$

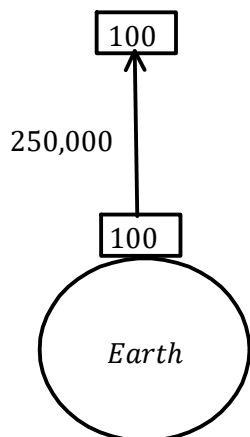
$$W = 4.89 \times 10^6 \text{ J}$$

$$W = GMm \left(\frac{1}{r_i} - \frac{1}{r_f} \right)$$

$$E_p = mgh$$

$$E_p = 100(9.8)(5000)$$

$$E_p = 4.9 \times 10^6 \text{ J}$$



$$W = \Delta E$$

$$W = \Delta E_k + \Delta E_p$$

$$W = \Delta E_p$$

$$W = E_{pf} - E_{pi}$$

$$W = -\frac{GMm}{R} - \left(-\frac{GMm}{r}\right)$$

$$W = -\frac{6.67E-11(5.98E24)(100)}{6.38E6 + 250000} - \left(-\frac{6.67E-11(5.98E24)(100)}{6.38E6}\right)$$

$$W = -6016078431 - -6251818182 \text{ J}$$

$$W = 235739751$$

$$W = 2.36 \times 10^8 \text{ J}$$

$$E_p \neq mgh; h > 10000 \text{ m}$$

$$E_{pi} + E_{ki} + \Delta E = E_{pf} + E_{kf}$$

$$E_{pi} + E_{ki} + W = E_{pf} + E_{kf}$$

$$W = (E_{pf} - E_{pi}) + (E_{kf} - E_{ki})$$

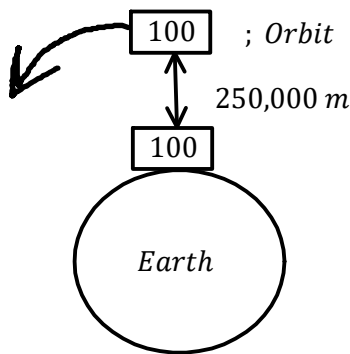
$$W = \Delta E$$

$$W = \Delta E$$

$$W = \Delta E_p + \Delta E_k$$

$$W = (E_{pf} - E_{pi}) + (E_{kf} - E_{ki})$$

P12 - 7.3 - Velocity in/Work to Orbit Notes



$$F_g = F_c \quad \boxed{F_c = F_g} \quad ; \text{Orbit}$$

$$\frac{GMm}{r^2} = ma_c$$

$$\frac{GMm}{r^2} = m \frac{v^2}{r} \quad a_c = \frac{v^2}{r}$$

$$\frac{GM}{r} = v^2 \quad ; \text{mass of object is irrelevant}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{6.67E-11 \times 5.98E24}{(6.38E6 + 250000)}}$$

$$v = 7756.338 \frac{m}{s}$$

Earth

$$E_p = -\frac{GMm}{r}$$

$$E_p = -\frac{6.67E-11(5.98E24)(100)}{6.38E6}$$

$$\boxed{E_p = -6.25E9 J}$$

$$E_k = \frac{1}{2} mv^2$$

$$\boxed{E_k = 0} \quad \text{To Change Orbit } v = \#$$

Orbit

$$E_p = -\frac{GMm}{r}$$

$$E_p = -\frac{6.67E-11(5.98E24)(100)}{6.38E6 + 250000}$$

$$\boxed{E_p = -6.01E9 J}$$

$$E_k = \frac{1}{2} mv^2$$

$$E_k = \frac{1}{2} (100)(7756.338)^2$$

$$\boxed{E_k = 3.01E9 J}$$

$$E_{pi} + E_{ki} + W = E_{pf} + E_{kf}$$

$$-6.25E9 + 0 + W = -6.01E9 + 3.01E9$$

$$W = \dots$$

Or

$$W = \Delta E$$

$$W = \Delta E_p + \Delta E_k \quad \boxed{; E_{ki} = 0}$$

$$W = (E_{pf} - E_{pi}) + (E_{kf} - E_{ki})$$

$$W = (-6.01E9 - (-6.25E9)) + (3.01E9 - 0)$$

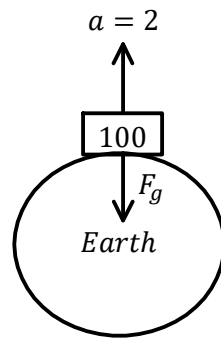
$$W = 3.26E9 J$$

$$W = GMm \left(\frac{1}{r_i} - \frac{1}{r_f} \right) + \left(\frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 \right)$$

$$W = \frac{GMm}{2} \left(\left(\frac{2}{r_i} - \frac{2}{r_f} \right) + (v_f^2 - v_i^2) \right)$$

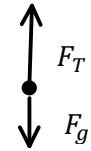
...

P12 - 7.4 - Dyn Thrust to Accelerate Notes



$$\begin{aligned}\Sigma F &= ma \\ F_T - F_g &= ma \\ F_T &= ma + mg \\ F_T &= m(a + g) \\ F_T &= 100(2 + 9.8) \\ F_T &= 1180 \text{ N}\end{aligned}$$

FBD



P12 - 7.5 - Pendulum Car Flat Circle Notes

F_c : Centripetal, The sum of the forces towards the centre minus the ones away (not in FBD!)

Find tension force at bottom.

2 kg
 $v = ? =$
 1 m
 1 m
 2 kg
 $v = 5 \frac{\text{m}}{2}$
 $F_g = mg$
 $F_g = 2 \times 9.8$
 $F_g = 19.6 \text{ N}$
 $T = ? = 69.6 \text{ N}$
 FBD
 T
 F_g
 $F_c = \frac{mv^2}{r}$
 $F_c = \frac{2(5)^2}{1}$
 $F_c = 50 \text{ N}$
 $F_c = T - F_g$
 $T = F_c + F_g$
 $T = 50 + 19.6$
 $T = 69.6 \text{ N}$

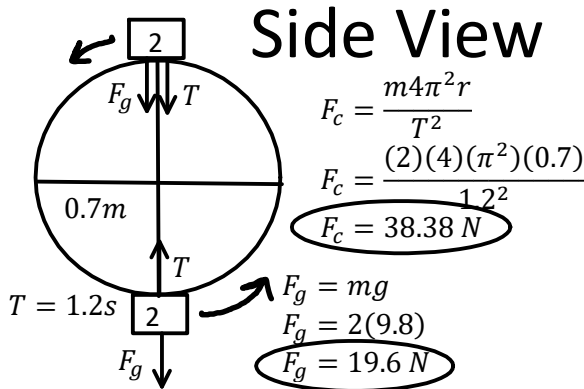
It takes 20 s for a car of $m = 2000\text{kg}$ to drive around a circular track with a $r = 15 \text{ m}$. Find v of the car. Find the "a" of the car. Find the Centripetal Force on the car.

Top View
 2000
 $r = 15$
 $T = 20\text{s}$
 $d = 2\pi r$
 $d = 2\pi(15)$
 $d = 30\pi \text{ m}$
 $d = 94.25 \text{ m}$
 $F_c = m \frac{4\pi^2 r}{T^2}$ **Or**
 $F_c = \frac{(2000)(4)\pi^2(15)}{20^2}$
 $F_c = 2960.88 \text{ N}$
 $v = \frac{d}{t}$
 $v = \frac{94.25}{20}$
 $v = 4.71 \frac{\text{m}}{\text{s}}$
 $a_c = \frac{v^2}{r}$
 $a_c = \frac{4.71^2}{15}$
 $a_c = 1.48 \frac{\text{m}}{\text{s}^2}$
 $F_c = ma_c$
 $F_c = (2000)(1.48)$
 $F_c = 2960.88 \text{ N}$
 What friction coefficient is required for the car not to slide off the track?
 $F_c = F_f$
 $ma_c = \mu mg$
 $\mu = \frac{a_c}{g}$
 $\mu = \frac{1.48}{9.8}$
 $\mu = 0.15$

P12 - 7.6 - Ball String Circle Notes

F_g : Straight down
 T : Towards centre
 F_n : Away from ground

A 2 kg mass on a 0.7 m string is spun around a circle with a period T of 1.2 s. Find the T in the string at the top and bottom of path.



Top

$$F_c = T + F_g$$

$$T = F_c - F_g$$

$$T = 38.38 - 19.6$$

$$T = 18.78 \text{ N}$$

Bottom

$$F_c = T - F_g$$

$$T = F_c + F_g$$

$$T = 38.38 + 19.6$$

$$T = 57.98 \text{ N}$$

What is the minimum v of the object at the top of the circular path to remain in circular motion?

$T = 0$

$$F_c = T + F_g$$

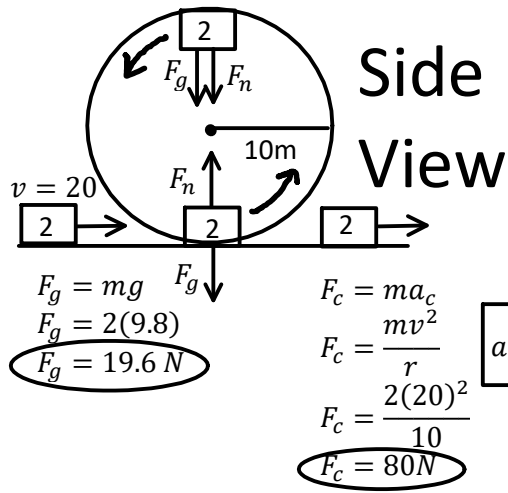
$$\frac{mv^2}{r} = mg$$

$$v = \sqrt{gr}$$

$$v = \sqrt{9.8(0.7)}$$

$$v = 2.62 \frac{m}{s}$$

A 2 kg cart follows a circular path $r = 10m$ with $v = 20 \frac{m}{s}$. Find F_n at the top and bottom.



Top

$$F_c = F_n + F_g$$

$$F_n = F_c - F_g$$

$$F_n = 80 - 19.6$$

$$F_n = 60.4 \text{ N}$$

Bottom

$$F_c = F_n - F_g$$

$$F_n = F_c + F_g$$

$$F_n = 80 + 19.6$$

$$F_n = 100.6 \text{ N}$$

What is the minimum velocity required for a cart not to fall off the circular path.

$F_n = 0$

$$F_c = T + F_g$$

$$F_c = F_g$$

$$\frac{mv^2}{r} = mg$$

$$\frac{v^2}{r} = g$$

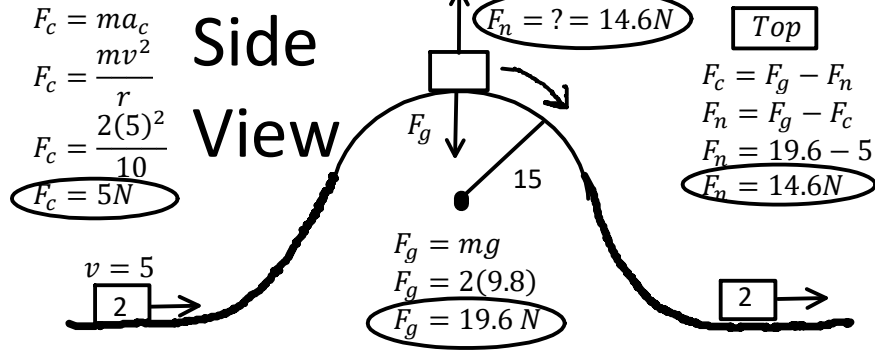
$$v = \sqrt{gr}$$

$$v = \sqrt{9.8(0.7)}$$

$$v = 2.62 \frac{m}{s}$$

P12 - 7.7 - Hills Notes

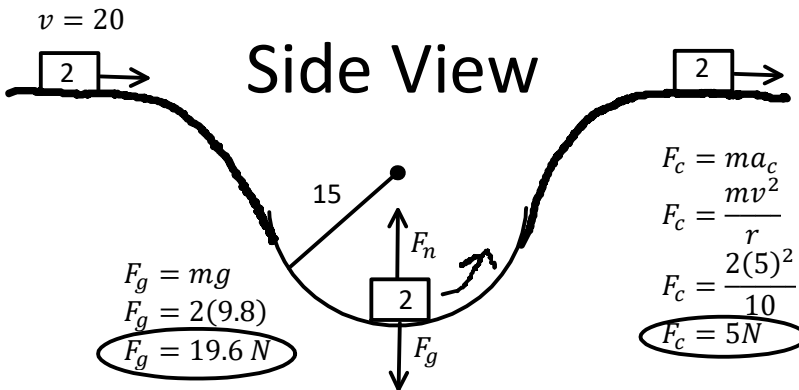
A 2kg cart goes over a circular hill of $r = 15$ at $5 \frac{m}{s}$. Find F_n at top.



Find the minimum v not to take air.

$F_c = F_g - F_n$
 ~~$\frac{mv^2}{r} = mg$~~ $F_n = 0$
 $v = \sqrt{gr}$
 $v = \sqrt{9.8(15)}$
 $v = 12.1 \frac{m}{s}$

A 2kg cart goes down a circular hill of $r = 15$ at $5 \frac{m}{s}$. Find F_n at bottom.

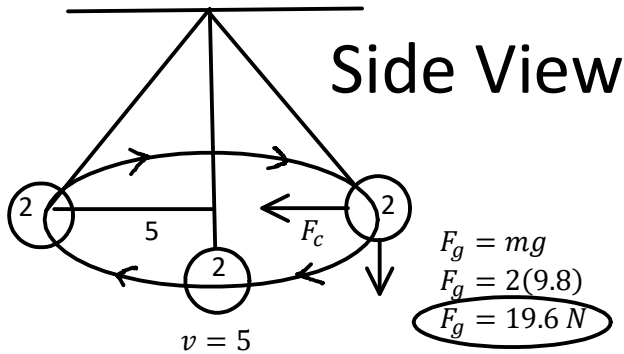


Bottom

$F_c = F_n - F_g$
 $F_n = F_c + F_g$
 $F_n = 80 + 19.6$
 $F_n = 99.6N$

P12 - 7.9 - Pendulum/Airplane Notes

A 2kg ball travels $5 \frac{m}{s}$ at in a circle $r = 1 m$ on a 2m string. Find T. ...



$$F_g = mg$$

$$F_g = 2(9.8)$$

$$F_g = 19.6 N$$

$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{2(5)^2}{1}$$

$$F_c = 10 N$$

~~$$E_{ki} + E_{pi} = E_{kf} + E_{pf}$$~~

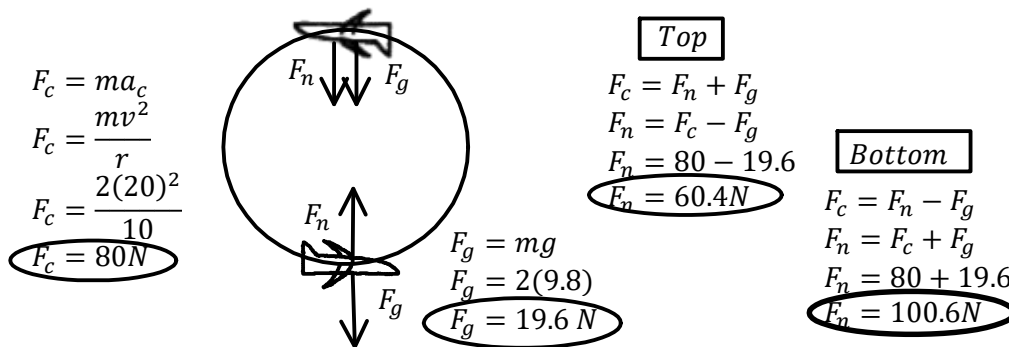
~~$$mgh = \frac{1}{2}mv_f^2$$~~

$$v_f = \sqrt{2gh}$$

$$v_f = \sqrt{(2)(9.8)(0.2)}$$

$$v_f = 1.98 \frac{m}{s}$$

A 2kg model plane travels $20 \frac{m}{s}$ at in a vertical circle $r = 10$. Find F_n on the pilot.



$$F_c = ma_c$$

$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{2(20)^2}{10}$$

$$F_c = 80 N$$

$$F_g = mg$$

$$F_g = 2(9.8)$$

$$F_g = 19.6 N$$

Top

$$F_c = F_n + F_g$$

$$F_n = F_c - F_g$$

$$F_n = 80 - 19.6$$

$$F_n = 60.4 N$$

Bottom

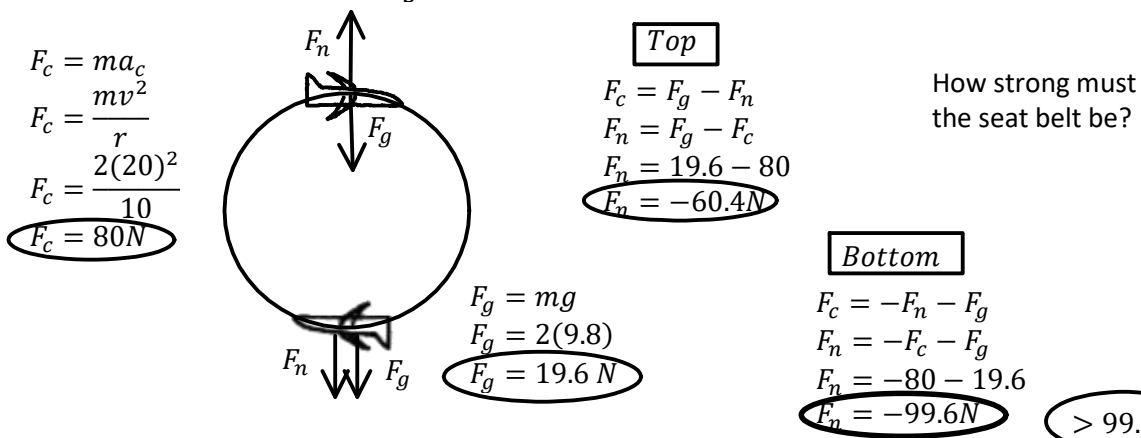
$$F_c = F_n - F_g$$

$$F_n = F_c + F_g$$

$$F_n = 80 + 19.6$$

$$F_n = 100.6 N$$

A 2kg model plane travels $20 \frac{m}{s}$ at in a vertical circle $r = 10$. Find F_n on the pilot.



$$F_c = ma_c$$

$$F_c = \frac{mv^2}{r}$$

$$F_c = \frac{2(20)^2}{10}$$

$$F_c = 80 N$$

$$F_g = mg$$

$$F_g = 2(9.8)$$

$$F_g = 19.6 N$$

Top

$$F_c = F_g - F_n$$

$$F_n = F_g - F_c$$

$$F_n = 19.6 - 80$$

$$F_n = -60.4 N$$

How strong must the seat belt be?

Bottom

$$F_c = -F_n - F_g$$

$$F_n = -F_c - F_g$$

$$F_n = -80 - 19.6$$

$$F_n = -99.6 N$$

$$> 99.6 N$$