

C12 - 1.2 - Limits Algebra Conj/LCD Notes

Find the Limits

$$\lim_{x \rightarrow 9} \frac{9-x}{3-\sqrt{x}}$$

$$\lim_{x \rightarrow 9} \frac{9-x}{3-\sqrt{x}} \times \frac{3+\sqrt{x}}{3+\sqrt{x}} \quad \leftarrow \text{Conjugate}$$

$$\lim_{x \rightarrow 9} \frac{(9-x)(3+\sqrt{x})}{9-x} \quad \text{Simplify} \quad \frac{(3-\sqrt{x})(3+\sqrt{x})}{9+3\sqrt{x}-3\sqrt{x}-x} = \frac{9-x}{9-x} \quad \boxed{\text{FL}}$$

$$\lim_{x \rightarrow 9} 3+\sqrt{x}$$

$$\frac{3+\sqrt{9}}{3+3} \quad \text{Substitute}$$

$$\boxed{6}$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{x+3} - \frac{1}{3}}{x} \quad LCD = 3(x+3)$$

$$\lim_{x \rightarrow 0} \frac{\frac{3-(x+3)}{3(x+3)}}{\frac{x}{1}} \quad \text{Add Fractions}$$

$$\lim_{x \rightarrow 0} \frac{\frac{-x}{3(x+3)}}{\frac{x}{1}} \quad \text{Simplify}$$

$$\lim_{x \rightarrow 0} \frac{-x}{3(x+3)} \times \frac{1}{x} \quad \text{Flip and Multiply}$$

$$\lim_{x \rightarrow 0} \frac{1}{-3(x+3)} \quad \text{Simplify}$$

$$\boxed{-\frac{1}{9}} \quad \text{Substitute}$$

$$\frac{\frac{1}{x+3} - \frac{1}{3}}{\frac{x}{1}}$$

OR

$$\frac{3-(x+3)}{3x(x+3)}$$

Multiply Top and Bottom by LCD

$$\frac{-x}{3x(x+3)}$$

LCD: 3(x+3)

$$\frac{-1}{3(x+3)}$$

C12 - 1.2 - Limits Trig Algebra Notes

$$\boxed{\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1}$$

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{2x} =$$

(1)

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin 2x}{x} \\ \lim_{x \rightarrow 0} \frac{\sin 2x}{x} \times \frac{2}{2} \\ \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \times 2 \\ 1 \times 2 \end{aligned}$$

(2)

$$\boxed{\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \times \frac{\sin x}{1 + \cos x}$$

$$1 \times \frac{0}{1+1}$$

(0)

$$\frac{1 - \cos x}{x} \times \frac{1 + \cos x}{1 + \cos x}$$

$$\frac{x(1 + \cos x)}{\sin^2 x} \times \frac{\sin x}{1 + \cos x}$$

Conjugate

Separate Fractions

$$\boxed{\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \times \frac{1}{\cos x}$$

$$1 \times \frac{1}{1}$$

(1)

$$\frac{\tan x}{x}$$

Proof

$$\frac{\cos x}{x}$$

Defn

$$\frac{1}{1} \times \frac{1}{x}$$

$$\frac{\sin x}{\cos x} \times \frac{1}{x}$$

$$\frac{\sin x}{x} \times \frac{1}{\cos x}$$

Flip and Multiply

$$\lim_{x \rightarrow 0} \frac{\tan 4x}{\tan 3x} \times \frac{4x}{4x}$$

$$\lim_{x \rightarrow 0} \frac{\tan 3x}{1} \times \frac{3x}{3x}$$

$$\lim_{x \rightarrow 0} \frac{\tan 4x}{4x} \times 4x$$

$$\lim_{x \rightarrow 0} \frac{\tan 3x}{3x} \times 3x$$

$$\frac{1 \times 4x}{1 \times 3x}$$

($\frac{4}{3}$)

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{4x}$$

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \times \frac{1}{2}$$

$$1 \times \frac{1}{2}$$

($\frac{1}{2}$)

Separate Product

$$4 = 2 \times 2$$

$$\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x} \times \frac{2x}{2x}$$

$$\lim_{x \rightarrow 0} \frac{1}{\sin 3x} \times \frac{2x}{3x}$$

$$\frac{1}{\sin 2x} \times \frac{2x}{2x}$$

$$\lim_{x \rightarrow 0} \frac{2x}{\sin 3x} \times \frac{1}{3x}$$

$$\frac{1}{1} \times \frac{2x}{3x}$$

($\frac{2}{3}$)