

C12 - 1.6 - e Limit Hmk

$$e = \sum_{k=0}^{\infty} \frac{1}{k!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \dots \approx 2.718 \dots$$

$$e = 2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 + \frac{1}{5 + \dots}}}}} \approx 2.718$$

$\lim_{x \rightarrow \pm\infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x \rightarrow 0} \left(1 + x\right)^{\frac{1}{x}} = e \approx 2.718$
 Must be reciprocals of each other. Make them!

Euler's Id
 $e^{i\pi} + 1 = 0$

$\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x =$
 $\lim_{x \rightarrow \infty} \left(\left(1 + \frac{2}{x}\right)^{\frac{x}{2}}\right)^2 =$ Exponent of 1
 Separate Fractions
 $\left(\left(1 + \frac{2}{x}\right)^{\frac{x}{2}}\right)^2 = e^2$

$\lim_{x \rightarrow -\infty} \left(1 - \frac{4}{x}\right)^x =$
 $\lim_{x \rightarrow -\infty} \left(\left(1 - \frac{4}{x}\right)^{\frac{-x}{4}}\right)^{-4} =$
 $\lim_{x \rightarrow -\infty} \left(\left(1 - \frac{4}{x}\right)^{\frac{-x}{4}}\right)^{-4} = e^{-4}$

$\lim_{x \rightarrow 0} \left(1 + \frac{x}{3}\right)^{\frac{1}{x}} =$
 $\lim_{x \rightarrow 0} \left(\left(1 + \frac{x}{3}\right)^{\frac{1}{x}}\right)^3 =$
 $\lim_{x \rightarrow 0} \left(\left(1 + \frac{x}{3}\right)^{\frac{3}{x}}\right)^3 = e^3$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x}\right)^x =$$

$$\lim_{x \rightarrow 0} \left(1 + \frac{x}{2}\right)^{\frac{1}{x}} =$$

$$\lim_{x \rightarrow \infty} \left(1 - \frac{5}{x}\right)^x =$$

$$\lim_{x \rightarrow 0} \left(1 + \frac{x}{3}\right)^{\frac{1}{x}} =$$

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{2x}\right)^x =$$

$$\lim_{x \rightarrow -\infty} \left(1 + \frac{4}{x}\right)^{-x} =$$