

C12 - 1.0 - Limits Properties Review

Limit: What y is approaching*.

$\lim_{x \rightarrow a^-} f(x)$ Left hand limit (LHL)
 $\lim_{x \rightarrow a^+} f(x)$ Right hand limit (RHL)

Limit Exists if and only if:

$$\begin{aligned} &LHL = RHL \\ &\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) \\ &\lim_{x \rightarrow a} f(x) = \text{Exists} \end{aligned}$$

Limit Does Not Exist if :

$$\begin{aligned} &LHL \neq RHL \\ &\lim_{x \rightarrow a^+} f(x) \neq \lim_{x \rightarrow a^-} f(x) \\ &\lim_{x \rightarrow a} f(x) = \text{Does not Exist} \end{aligned}$$

Continuous : Limit exists and equals the value of the function. Obviously!

$$\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x) = f(a) \quad \text{OR} \quad \lim_{x \rightarrow a} f(x) = f(a)$$

Differentiable :
 LH Derivative equals
 RH Derivative

Given

$$\begin{aligned} \lim_{x \rightarrow c} f(x) &= L && \text{The Limit of } f(x), \\ &&& \text{as } x \text{ approaches } c, \text{ equals } L \\ \lim_{x \rightarrow c} g(x) &= M && \text{The Limit of } g(x), \\ &&& \text{as } x \text{ approaches } c, \text{ equals } M \end{aligned}$$

Examples

$$\lim_{x \rightarrow 2} 3x = 6$$

$$\lim_{x \rightarrow 2} x^2 = 4$$

Sum Rule

$$\lim_{x \rightarrow c} (f(x) + g(x)) = L + M$$

$$\lim_{x \rightarrow 2} 3x + x^2 = 6 + 4 = 10$$

Difference Rule

$$\lim_{x \rightarrow c} (f(x) - g(x)) = L - M$$

$$\lim_{x \rightarrow 2} 3x - x^2 = 6 - 4 = 2$$

Constant Rule

$$\lim_{x \rightarrow c} (k \times g(x)) = k \times M$$

$$\lim_{x \rightarrow 2} 5x^2 = 5 \times 4 = 20$$

Product Rule

$$\lim_{x \rightarrow c} (f(x) \times g(x)) = L \times M$$

$$\lim_{x \rightarrow 2} 3x \times x^2 = 6 \times 4 = 24$$

Quotient Rule

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{M}$$

$$\lim_{x \rightarrow 2} \frac{3x}{x^2} = \frac{6}{4}$$

Power Rule

$$\lim_{x \rightarrow c} (f(x))^{\frac{m}{n}} = \left(\frac{L}{M}\right)^{\frac{m}{n}}$$

$$\lim_{x \rightarrow 2} (3x)^{\frac{1}{2}} = \left(\frac{6}{2}\right)^{\frac{1}{2}}$$

Separate Limits

$$\lim_{x \rightarrow c} f(x) + g(x) =$$

$$\lim_{x \rightarrow c} f(x) + \lim_{x \rightarrow c} g(x)$$