C12 - 2.10 - IVT/MVT/LinApp/NewMeth/TaySer Notes

IVT

Is this Possible?

$3 = x^2$?

$0 = x^2 - 3$ Get = 0

$y = x^2 - 3$ Function

	x	y
a	1	-2
c	1.73	0
b	2	1

See Graph Below

$+ve \rightarrow -ve$ OR $-ve \rightarrow +ve$

$f(a) \le f(c) \le f(b)$

$f(1) \le f(c) \le f(2)$

$-2 \le 0 \le 1$

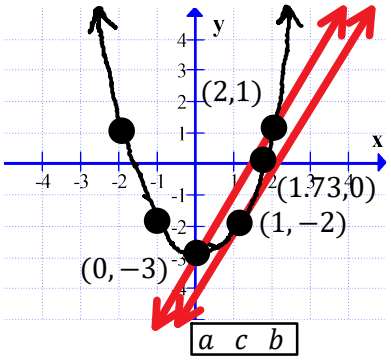
There is a value $x = c$ between a and b ; where $f(x) = 0$, (& Continuous [a,b]) therefor $x^2 = 3$ must have a solution. True.

These should be obvious to you!

MVT

$f(x) = x^2 - 3$

Tangent Slope = Secant Slope



$y' = m$

$y' = \frac{y_2 - y_1}{x_2 - x_1}$

$2x = \frac{1 - -3}{2 - 0}$

$2x = 2$

$x = 1$

$y = x^2 - 3$

$y' = 2x$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{1 - -3}{2 - 0}$

$m = 2$

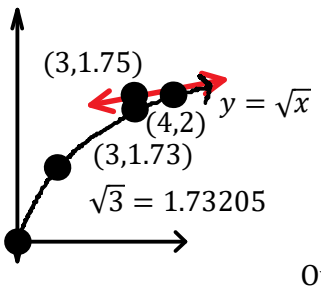
$0 \le x \le 2$ [0,2]

x	y
0	-3
2	1

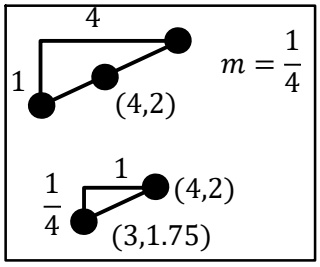
There is a value $x = c$ between a and b ; where $f'(c) = \frac{f(b) - f(a)}{b - a}$,

(& Continuous [a,b] & Differentiable [a,b].)

$\sqrt{3} = ?$ $y = \sqrt{x}$ $(3, y)$ $y = ?$ $\sqrt{4} = 2$



$\sqrt{3} = 1.73205$



Overestimate (concave down.)

Slope × horizontal distance = height

$y = \sqrt{x}$

$y' = \frac{1}{2\sqrt{x}}$

$m = \frac{1}{2\sqrt{4}}$

$m = \frac{1}{4}$

$y - y_1 = m(x - x_1)$

$L(x) = y_1 + m(x - x_1)$

$L(x) = 2 + \frac{1}{4}(3 - 4)$

$L(x) = 2 - \frac{1}{4} \approx 1.75$

Linear Approximation

$3 = x^2$ $\sqrt{3} = ?$

$0 = x^2 - 3$ Get = 0

$f(x) = x^2 - 3$ Function

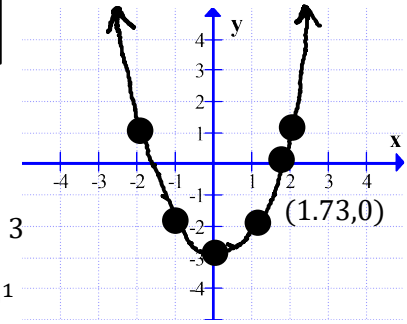
$x_1 = 2$ Estimate OR 1*

$y - y_1 = m(x - x_1)$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Newton's Method

$f(x) = x^2 - 3$



Calc

2* Store x Enter

$x - \frac{(x^2 - 3)}{(2x^1)}$

Repeat

Store x Enter

^^^^ Upx4

Repeat Last 2 Steps*

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$x_2 = 2 - \frac{f(2)}{f'(2)}$

$f(x) = x^2 - 3$ $f(2) = (2)^2 - 3$ $f(2) = 1$

$f'(x) = 2x^1$ $f'(2) = 2(2)^1$ $f'(2) = 4$

$x_2 = 2 - \frac{1}{4}$

$x_2 = 1.75$

REPEAT

$x_3 = 1.73214$

$x_4 = 1.73205$

Check

$\sqrt{3} = 1.73205$

Taylor Series Poly

$\sqrt{3} = ?$ $f(x) = \sqrt{x}$

$a = 4$

$x = 3$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(4) = \frac{1}{2\sqrt{4}}$

$= \frac{1}{4}$

$T(x) = f(a) + \frac{f'(a)(x-a)^1}{1!} + \frac{f''(x)(x-a)^2}{2!} \ldots$

$T(x) = f(4) + \frac{1(x-4)^1}{4 \times 1} - \frac{1(x-4)^2}{32 \times 2} \ldots$

$T(x) = 2 + \frac{(x-4)}{4} - \frac{(x-4)^2}{64} \ldots$

$T(3) = 2 + \frac{(3-4)}{4} - \frac{(3-4)^2}{64} = 1.734$

$f'(x) = \frac{1x^{-\frac{1}{2}}}{2}$

$f''(x) = -\frac{1}{4\sqrt{x^3}}$

$f''(4) = -\frac{1}{4\sqrt{4^3}}$

$= -\frac{1}{32}$