C12 - 2.11 - Ln Derivatives Notes

$y' = \frac{dy}{dx} = f'(x)$

Log Rules

$y = lnx$
$y' = \frac{1}{xlne}$
$y' = \frac{1}{x}$

$y = \log_5 x$
$y' = \frac{1}{xln5}$

$y = ln2x$
$y' = \frac{1}{2x} \times 2$
$y' = \frac{1}{x}$

$y = \log_5 2x$
$y' = \frac{1}{2x}\frac{1}{ln5}(2)$
$y' = \frac{1}{xln5}$

$y = \log_7 x^2$
$y' = \frac{1}{x^2 ln7} \times 2x$
$y' = \frac{2}{xln7}$

$y = \log_7 x^2$
$y = 2\log_7 x$
$y' = 2 \times \frac{1}{xln7}$
$y' = \frac{2}{xln7}$

$y = \log_5 x^{\frac{1}{2}}$
$y = \frac{1}{2}\log_5 x$
$y' = \frac{1}{2}(\frac{1}{xln5})$
$y' = \frac{1}{2xln5}$

$y = \ln(that)$
$y' = \frac{chain\ that}{that}$

$y = \log_b(that)$
$y' = \frac{chain\ that}{that(lnb)}$

$y = \ln(lnx)$
$y' = \frac{1}{lnx} \times \frac{1}{x}$
$y' = \frac{1}{xlnx}$

$y = \ln(x^2)$
$y' = \frac{1}{x^2} \times 2x$
$y' = \frac{2}{x}$

$y = \ln(x^2)$
$y = 2lnx$
$y' = 2 \times \frac{1}{x}$
$y' = \frac{2}{x}$

$y = \ln(1 + x^2)$
$y' = \frac{1}{1+x^2} \times 2x$
$y' = \frac{2x}{1+x^2}$

$y = \ln(2x + 3)$
$y' = \frac{1}{2x+3} \times 2$
$y' = \frac{2}{2x+3}$

$y = \log_2(3x + 1)$
$y' = \frac{1}{(3x+1)ln2}(3)$
$y' = \frac{3}{(3x+1)ln2}$

$y = (lnx)^2$
$y' = 2(lnx)^1 \times \frac{1}{x}$
$y' = \frac{2lnx}{x}$

$y = \ln(x\sqrt{x-1})$
$y = lnx + ln\sqrt{x-1}$
$y' = \frac{1}{x} + \frac{1}{\sqrt{x-1}}\left(\frac{1}{2\sqrt{x-1}}\right)$
$y' = \frac{1}{x} + \frac{1}{2x-2}$
$y' = \frac{3x-2}{x(2x-2)}$

$y = \ln(\frac{x+1}{x-1})$
$y = \ln(x+1) - \ln(x-1)$
$y' = \frac{1}{x+1} - \frac{1}{x-1}$
$y' = -\frac{2}{(x^2-1)}$

$y = logx^{logx}$
$y = logxlogx$
$y = (logx)^2$
$y' = 2(logx)^1 \times \frac{1}{xln10}$
$y' = \frac{2logx}{xln10}$

$y = xlnx$
$y' = 1(lnx) + \frac{1}{x} \times x$
$y' = lnx + 1$

$y = lnx^x$
$y = xlnx$
$y' = lnx + 1$

$y = lnxlnx$
$y' = \frac{1}{x}lnx + \frac{1}{x} \times lnx$
$y' = \frac{2lnx}{x}$

$\boldsymbol{y = x^x}$ Ln Both Sides
$lny = lnx^x$
$lny = xlnx$
$\frac{1}{y} \times y' = 1(lnx) + \frac{1}{x} \times x$
$\frac{y'}{y} = lnx + 1$
$y' = y(lnx + 1)$
$y' = x^x(lnx + 1)$

$y = x^{lnx}$
$lny = lnx^{lnx}$
$lny = lnxlnx$
$\frac{y'}{y} = \frac{1}{x}lnx + \frac{1}{x} \times lnx$
$\frac{y'}{y} = \frac{lnx}{x} + \frac{lnx}{x}$
$\frac{y'}{y} = \frac{2lnx}{x}$
$y' = y\left(\frac{2lnx}{x}\right)$
$y' = x^{lnx}\left(\frac{2lnx}{x}\right)$

A difficult quotient

$y = \frac{(2x+1)^2}{(x+2)^3}$
$lny = ln\frac{(2x+1)^2}{(x+2)^3}$
$lny = \ln(2x+1)^2 - \ln(x+2)^3$
$lny = 2\ln(2x+1) - 3\ln(x+2)$
$\frac{y'}{y} = 2 \times \frac{1}{2x+1} \times 2 - 3 \times \frac{1}{x+2}$
$y' = y\left(2 \times \frac{1}{2x+1} \times 2 - 3 \times \frac{1}{x+2}\right)$
$y' = \frac{(2x+1)^2}{(x+2)^3}\left(\frac{4}{2x+1} - \frac{3}{x+2}\right)$
$y' = \frac{(2x+1)^2}{(x+2)^3}\left(\frac{5-2x}{(2x+1)(x+2)}\right)$
$y' = \frac{(2x+1)(5-2x)}{(x+2)^4}$

$y = \ln(sinx)$
$y' = \frac{1}{sinx}(cosx)$
$y' = cotx$

$y = \ln(xsinx)$
$y' = \frac{1}{xsinx}(1(sinx) + (-cosx)(x))$
$y' = \frac{sinx - xcosx}{xsinx}$