C12 - 2.13 - Implicit Derivatives Notes

$y = x^3$ $\boxed{\text{Chain Rule}}$

$$\frac{dy}{dx} = 3x^2 \times \frac{dx}{dx}$$

$$\frac{dy}{dx} = 3x^2 \times 1$$

$y = y^3$ $\boxed{\text{Chain Rule}}$

$$\frac{dy}{dx} = 3y^2 \times \frac{dy}{dx}$$

$$\frac{dy}{dx} = 3y^2 \times y'$$

$$2x = 3y$$
$$\frac{2x}{3} = y$$
$$y' = \frac{2}{3}$$

$$xy = 1$$
$$1(y) + y'(x) = 0$$
$$y'x = -y$$
$$y' = -\frac{y}{x}$$

...

$$y = \frac{1}{x}$$
$$y = x^{-1}$$
$$y' = -\frac{1}{x^2}$$

$\boxed{\text{Not Implicit}}$

$\boxed{-2xy}$

$$-2xy$$
$$-2(1y + y'x)$$
$$-2xy$$
$$-2y + y'(-2x)$$

$$xy = 0$$
$$1y + y'x = 0$$
$$y'x = -y$$
$$y' = -\frac{y}{x}$$

$$y^2 = x$$
$$2yy' = 1$$
$$y' = \frac{1}{2y}$$

$$y^2 = x$$
$$y = \pm\sqrt{x}$$
$$y' = \frac{\pm 1}{2\sqrt{x}}$$

$$xy^2 = 2$$
$$1(y^2) + 2yy'(x) = 0$$
$$y^2 + 2xyy' = 0$$
$$2xyy' = -y^2$$
$$y' = \frac{-y^2}{2xy}$$

$$y = \sqrt{xy}$$
$$y^2 = xy$$
$$2yy' = (1(y) + y'(x))$$
$$2yy' = y + xy'$$
$$2yy' - xy' = y$$
$$y'(2y - x) = y$$
$$y' = \frac{y}{2y - x}$$

$$x^2 + xy + y^2 = 9$$
$$2x + \big(1(y) + y'(x)\big) + 2yy' = 0$$
$$2x + y + xy' + 2yy' = 0$$
$$xy' + 2yy' = -2x - y$$
$$y'(x + 2y) = -2x - y$$
$$y' = \frac{-2x - y}{x + 2y}$$
$$y' = -\frac{2x + y}{x + 2y}$$

$$x^2 + y^2 = 9$$
$$2x + 2yy' = 0$$
$$2yy' = -2x$$
$$y' = -\frac{x}{y}$$
$$y' = \frac{\pm x}{\sqrt{9 - x^2}}$$

$$x^2 + y^2 = 9$$
$$y = \pm\sqrt{9 - x^2}$$
$$y' = \frac{\pm x}{\sqrt{9 - x^2}}$$

$$y' = -\frac{x}{y}$$
$$y'' = -\left(\frac{1y - y'x}{y^2}\right)$$
$$y'' = -\left(\frac{1y - (-\frac{x}{y})x}{y^2}\right)$$
$$y'' = -\left(\frac{1y^2 + x^2}{y^3}\right)$$
$$y'' = -\left(\frac{9}{y^3}\right)$$
$$y'' = \frac{\pm 9}{(9 - x^2)^{\frac{3}{2}}}$$

$$3cosx + siny = x^2$$
$$-3simx + cosyy' = 2x$$
$$y' = \frac{2x + 3sinx}{cosy}$$

$$y = \cos(x + y)$$
$$y' = -sin(x + y)(1 + y')$$
$$y' = -\sin(x + y) - y'\sin(x + y)$$
$$y' - y'\sin(x + y) = -\sin(x + y)$$
$$y'(1 - \sin(x + y)) = -\sin(x + y)$$
$$y' = \frac{-\sin(x + y)}{1 - \sin(x + y)}$$

$$xcosy = \sin(x + y)$$
$$cosy - xy'siny = \cos(x + y)(1 + y')$$
$$cosy - xy'siny = 1\cos(x + y) + y'\cos(x + y)$$
$$cosy - 1\cos(x + y) = y'\cos(x + y) + xy'siny$$
$$cosy - 1\cos(x + y) = y'(\cos(x + y) + xsiny)$$
$$y' = \frac{cosy - 1\cos(x + y)}{\cos(x + y) + xsiny}$$