C12 - 2.14 - Functions/Inverse Derivative Notes

$y = f(x)$
$y' = f'(x) \times 1$

$y = f(x^3)$
$y' = f'(x^3) \times 3x^2$

$y = f(that)$
$y' = f'(that) \times (chain\ that)$

$y = f^2(x)$
$y = \big(f(x)\big)^2$
$y' = 2\big(f(x)\big)^1 f'(x)$
$y' = 2f(x)f'(x)$

$y = f^3(2x)$
$y = \big(f(2x)\big)^3$
$y' = 3\big(f(2x)\big)^2 f'(2x) \times 2$
$y' = 6f^2(2x)f'(2x)$

$y = \sqrt{f(x)}$
$y = \big(f(x)\big)^{\frac{1}{2}}$
$y' = \frac{1}{2} f(x)^{-\frac{1}{2}} \times f'(x)$
$y' = \frac{f'(x)}{2\sqrt{f(x)}}$

$y = e^{f(x)}$
$y' = e^{f(x)} f'(x)$

Find $(f^{-1})'(x)$ *of* $f(x) = x^3 + 1$, @ $y = 9$.

$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$
$(f^{-1})'(x) = \frac{1}{3(x-1)^{\frac{2}{3}}}$
$(f^{-1})'(9) = \frac{1}{3(9-1)^{\frac{2}{3}}}$
$(f^{-1})'(9) = \frac{1}{12}$

$f(x) = x^3 + 1$
$y = x^3 + 1$
$x = y^3 + 1$
$y = \sqrt[3]{x-1}$
$f^{-1}(x) = \sqrt[3]{x-1}$

$f^{-1}(x) = \sqrt[3]{x-1}$
$f^{-1}(x) = (x-1)^{\frac{1}{3}}$
$f^{-1}(x) = \frac{1}{3}(x-1)^{-\frac{2}{3}}$
$(f^{-1})'(x) = \frac{1}{3(x-1)^{\frac{2}{3}}}$

$f(x) = x^3 + 1$ $\quad y = x^3 + 1$
$f'(x) = 3x^2$ $\quad 9 = x^3 + 1$
$f'(2) = 3(2)^2$ $\quad x = 2$ $\quad (2,9)$
$f'(2) = 12$

$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$
$(f^{-1})'(x) = \frac{1}{f'(2)} = \frac{1}{12}$

$h(x) = f\big(g(x)\big)$
$h'(x) = f'\big(g(x)\big) \times g'(x)$

$g(x) = x^2 f(x)$
$g'(x) = 2xf(x) + x^2 f'(x)$

$f(x) = \frac{g(x)}{\sqrt{x}}$
$f'(x) = \frac{g'(x)\sqrt{x} - \frac{1}{2\sqrt{x}} g(x)}{x}$
$f'(x) = \frac{g'(x)2x - g(x)}{2x\sqrt{x}}$

$f(x) = g(x^{-1})$
$f'(x) = g'(x^{-1})\left(-\frac{1}{x^2}\right)$
$f'(x) = \frac{-g'(x^{-1})}{x^2}$

$h(x) = g\big(g(x)\big)$
$h'(x) = g'\big(g(x)\big) \times g'(x)$