C12 - 2.15 - Trig Derivatives Notes

$y = sin2x$
$y' = cos2x \times 2$
$y' = 2cos2x$

$y = cosx^2$
$y' = -sinx^2 \times 2x$
$y' = -2x(sinx^2)$

$y = \sin(cosx)$
$y' = \cos(cosx) \times (-sinx)$
$y' = -sinxcos^2x$

Trig Identities

$y = sin2x$
$y = 2sinxcosx$
$y' = 2(cosxcosx + (-sinxsinx))$
$y' = 2(\cos^2 x - \sin^2 x)$
$y' = 2cos2x$

$y = \sin(that)$
$y' = \cos(that) \times (\text{chain that})$

$y = \cos(that)$
$y' = -\sin(that) \times (chain\ that)$

$y = \sin(3x+1)$
$y' = \cos(3x+1) \times 3$
$y' = 3\cos(3x+1)$

$y = \sin^2 x$
$y = (sinx)^2$
$y' = 2sinx \times cosx$
$y' = 2sinxcosx$
$y' = sin2x$

$y = \sin^2 3x$
$y = (sin3x)^2$
$y' = 2(sin3x)(cos3x)(3)$
$y' = 6sin3xcos3x$
$y' = 3sin6x$

Inside

$y = sin3x$
$y' = cos3x(3)$

Not a Product

$y = sinxsecx$
$y = sinx\dfrac{1}{cosx}$
$y = tanx$
$y' = \sec^2 x$

$y = \sec(2x)$
$y' = \sec(2x)\tan(2x) \times 2$
$y' = 2\sec(2x)\tan(2x)$

$y = cscx^2$
$y' = -cscx^2cotx^2(2x)$

$y = \sec(tanx)$
$y' = \sec(tanx)\tan(tanx) \times \sec^2 x$

$y = \tan(\sqrt{x})$
$y' = \sec^2\sqrt{x}\left(\dfrac{1}{2\sqrt{x}}\right)$
$y' = \dfrac{\sec^2\sqrt{x}}{2\sqrt{x}}$

$y = \cot(\frac{1}{x})$
$y' = -\csc^2\left(\dfrac{1}{x}\right)\left(\dfrac{-1}{x^2}\right)$
$y' = \dfrac{\csc^2(\frac{1}{x})}{x^2}$

Proofs

$y = \dfrac{1}{cosx} = secx$
$y' = \dfrac{0(cosx) - -sinx(1)}{\cos^2 x}$
$y' = \dfrac{sinx}{\cos^2 x}$
$y' = \dfrac{1}{cosx} \times \dfrac{sinx}{cosx}$
$y' = secxtanx$

$y = \dfrac{sinx}{cosx} = tanx$
$y' = \dfrac{\cos^2 x + \sin^2 x}{\cos^2 x}$
$y' = \dfrac{1}{\cos^2 x} = \sec^2 x$

$y = \tan(that)$
$y' = \sec^2(that) \times chain\ that$

$y = \dfrac{1}{sinx} = cscx$
$y' = \dfrac{0(sinx) - (\cos x)(1)}{\sin^2 x}$
$y' = \dfrac{-cosx}{\sin^2 x}$
$y' = \dfrac{1}{sinx} \times \dfrac{-cosx}{sinx}$
$y' = -cscxcotx$

$y = \dfrac{cosx}{sinx} = cotx$
$y' = \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x}$
$y' = \dfrac{-1}{\sin^2 x} = -\csc^2 x$

$y = \cot(that)$
$y' = -\csc^2(that) \times (chain\ that)$

$y = \sec(that)$
$y' = \sec(that)\tan(that) \times (chain\ that)$

$y = \csc(that)$
$y' = -\csc(that)\cot(that) \times (chain\ that)$

$y = \cos(sin2x)$
$y' = -\sin(sin2x)(cos2x)(2)$

$y = \cos^3(sin2x)$
$y = (\cos(sin2x))^3$
$y' = 3(\cos(sin2x))^2(-\sin(sin2x)(cos2x)(2))$
$y' = -6(\cos(sin2x))^2\sin(sin2x)\cos 2x$

Inside

$y = \cos(sin2x)$
$y' = -\sin(sin2x)(cos2x)(2)$

$y = \sqrt{cos5x}$
$y = (cos5x)^{\frac{1}{2}}$
$y' = \dfrac{1}{2}(cos5x)^{-\frac{1}{2}}(-sin5x)(5)$
$y' = -\dfrac{5sin5x}{2\sqrt{cos5x}}$

Inside

$y = cos5x$
$y' = -\sin(5x)(5)$

$y = 2\sec^2 x^7$
$y = 2(secx^7)^2$
$y' = 4(secx^7)^1(secx^7)(tanx^7)(7x^6)$
$y' = 28x^6(\sec^2 x^7)(tanx^7)$

Inside

$y = secx^7$
$y' = (secx^7)(tanx^7)(7x^6)$

$y = xsinx$
$y' = sinx + xcosx$

C12 - 2.15 - Trig Derivative Proofs

$\sin(a+b) = sinacosb + sinbcosa$
$\cos(a+b) = cosacosb - sinasinb$

Definition.
Trig IDs.
Separate Fractions.
Factor.

$f(x) = sinx$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{sin(x+h)-sinx}{h}$$

$$\lim_{h\to 0} \frac{sinxcosh + sinhcosx - sinx}{h}$$

$$\lim_{h\to 0} \frac{sinxcosh - sinx}{h} + \lim_{h\to 0} \frac{sinhcosx}{h}$$

$$\lim_{h\to 0} \frac{sinx(cosh-1)}{h} + \frac{sinhcosx}{h}$$

$$\lim_{h\to 0} sinx\frac{(cosh-1)}{h} + \lim_{h\to 0} \frac{sinh}{h} \times cosx$$

$$sinx \times 0 + 1 \times cosx$$

$$f'(x) = cosx$$

$$\lim_{x\to 0} \frac{sinx}{x} = 1$$

$f(x) = cosx$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{cos(x+h)-cosx}{h}$$

$$\lim_{h\to 0} \frac{cosxcosh - sinxsinh - cosx}{h}$$

$$\lim_{h\to 0} \frac{cosxcosh - cosx}{h} - \lim_{h\to 0} \frac{sinxsinh}{h}$$

$$\lim_{h\to 0} \frac{cosx(cosh-1)}{h} - \lim_{h\to 0} \frac{sinxsinh}{h}$$

$$\lim_{h\to 0} cosx\frac{(cosh-1)}{h} - \lim_{h\to 0} sinx \times \frac{sinh}{h}$$

$$cosx \times 0 - sinx \times 1$$

$$f'(x) = -sinx$$

C12 - 2.15 - Trig Derivatives Notes

$$y = \frac{cotx}{1 + cscx}$$
$$y = \frac{\frac{cosx}{sinx}}{1 + \frac{1}{sinx}} \begin{matrix} \times LCD \\ \times LCD \end{matrix}$$
$$y = \frac{cosx}{sinx + 1}$$
$$y' = \frac{\sin^2 x + cosx + \cos^2 x}{(sinx + 1)^2}$$
$$y' = \frac{1 + cosx}{(sinx + 1)^2}$$

$$y = cscxcotx = \frac{cscx}{tanx}$$
$$y = \frac{1}{sinx}\frac{cosx}{sinx}$$
$$y = \frac{cosx}{\sin^2 x}$$
$$y' = \frac{-\sin^3 x - 2sinxcos^2x}{\sin^4 x}$$
$$y' = \frac{-\sin^2 x - 2cos^2x}{\sin^3 x}$$
$$y' = \frac{-\sin^2 x - 2(1 - \sin^2 x)}{\sin^3 x}$$
$$y' = \frac{\sin^2 x - 2}{\sin^3 x} = cscx - 2\csc^3 x$$

$$y = cscxcotx$$
$$y' = -cscxcot^2x - \csc^3 x$$
$$y' = -\frac{1}{sinx}\frac{\cos^2 x}{\sin^2 x} - \frac{1}{\sin^3 x}$$
$$y' = \frac{-\cos^2 x - 1}{\sin^3 x}$$
$$y' = \frac{\sin^2 x - 2}{\sin^3 x}$$
$$y' = cscx - 2\csc^3 x$$

$$y = \left(\frac{sinx}{1 + cosx}\right)^2$$
$$y' = 2\left(\frac{sinx}{1 + cosx}\right)^1\left(\frac{1}{1 + cosx}\right)$$
$$y' = \frac{2sinx}{(1 + cosx)^2}$$

Inside

$$y = \frac{sinx}{1 + cosx}$$
$$y' = \frac{(cosx)(1 + cosx) - (-sinx)(sinx)}{(1 + cosx)^2}$$
$$y' = \frac{cosx + \cos^2 x + \sin^2 x}{(1 + cosx)^2}$$
$$y' = \frac{cosx + \cos^2 x + \sin^2 x}{(1 + cosx)^2}$$
$$y' = \frac{cosx + 1}{(1 + cosx)^2}$$
$$y' = \frac{1}{1 + cosx}$$

$u = sinx$
$u' = cosx$

$v = 1 + cosx$
$v' = -sinx$

$$y = \frac{u}{v}$$
$$y' = \frac{u'v - v'u}{v^2}$$

$$y = \sin(xy)$$
$$y' = \cos(xy)(1y + y'x)$$

C12 - 2.15 - Inverse Trig Derivative Notes

$y=\tan^{-1}x^3$

$y'=\dfrac{1}{1+(x^3)^2}\times 3x^2$

$y'=\dfrac{3x^2}{1+x^6}$

$y=\sin^{-1}(2x)$

$y'=\dfrac{1}{\sqrt{1-(2x)^2}}(2)$

$y'=\dfrac{2}{\sqrt{1-4x^2}}$

$y=\cos^{-1}(x^2)$

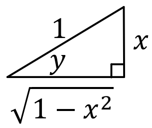
$y'=-\dfrac{1}{\sqrt{1-(x^2)^2}}(2x)$

$y'=-\dfrac{2x}{\sqrt{1-x^4}}$

$y=\sec^{-1}(2x)$

$y'=\dfrac{1}{2x\sqrt{(2x)^2-1}}(2)$

$y'=\dfrac{1}{x\sqrt{4x^2-1}}$



$siny=x$

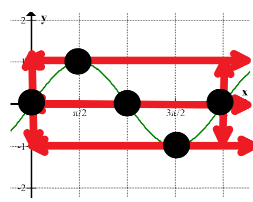
$cosy\,y'=1$

$y'=\dfrac{1}{cosy}$

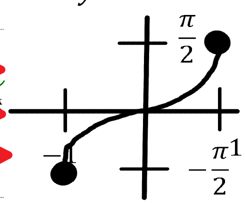
$y'=\dfrac{1}{\sqrt{1-x^2}}$

$cosy=\sqrt{1-x^2}$

$y=\sin x$

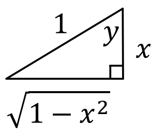


$y=\sin^{-1}x$



Range

$-\dfrac{\pi}{2}\le y\le\dfrac{\pi}{2}$



$cosy=x$

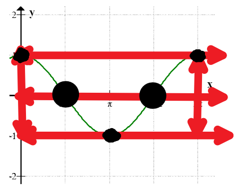
$-siny\,y'=1$

$y'=-\dfrac{1}{siny}$

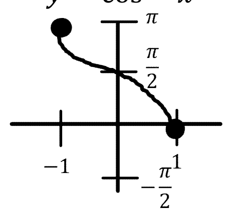
$y'=\dfrac{-1}{\sqrt{1-x^2}}$

$siny=\sqrt{1-x^2}$

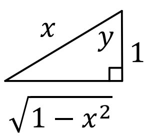
$y=\cos x$



$y=\cos^{-1}x$



$0\le y\le\pi$



$secy=x$

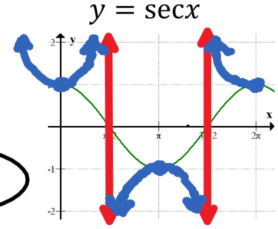
$secytany\,y'=1$

$y'=\dfrac{1}{secytany}$

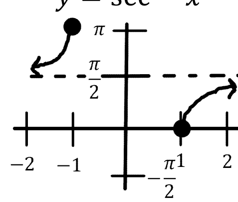
$y'=\dfrac{1}{|x|\sqrt{x^2-1}}$

$tany=\sqrt{1-x^2}$

$y=secx$

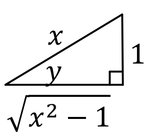


$y=\sec^{-1}x$



$0<y<\dfrac{\pi}{2}$

$\dfrac{\pi}{2}<y<\pi$



$cscy=x$

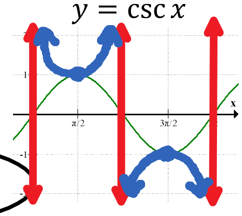
$-cscycotyy\,y'=1$

$y'=\dfrac{-1}{cscycoty}$

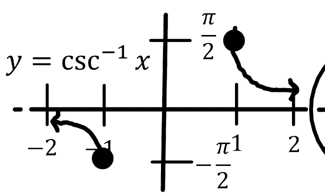
$y'=\dfrac{-1}{|x|\sqrt{x^2-1}}$

$coty=\sqrt{x^2-1}$

$y=\csc x$

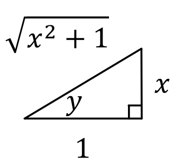


$y=\csc^{-1}x$



$0<y<\dfrac{\pi}{2}$

$-\dfrac{\pi}{2}<y<0$



$tany=x$

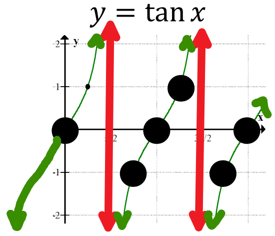
$\sec^2 y\,y'=1$

$y'=\dfrac{1}{\sec^2 y}$

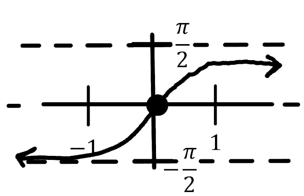
$y'=\dfrac{1}{x^2+1}$

$secy=\sqrt{x^2+1}$

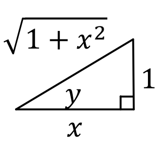
$y=\tan x$



$y=\tan^{-1}x$



$-\dfrac{\pi}{2}<y<\dfrac{\pi}{2}$



$coty=x$

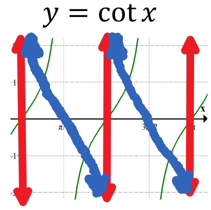
$-\csc^2 y\,y'=1$

$y'=\dfrac{-1}{\csc^2 y}$

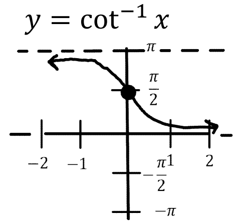
$y'=\dfrac{-1}{1+x^2}$

$cscy=\sqrt{1+x^2}$

$y=\cot x$



$y=\cot^{-1}x$



$0\le y\le\pi$