C12 - 2.3 - Definition of Derivative Equation Graph Notes

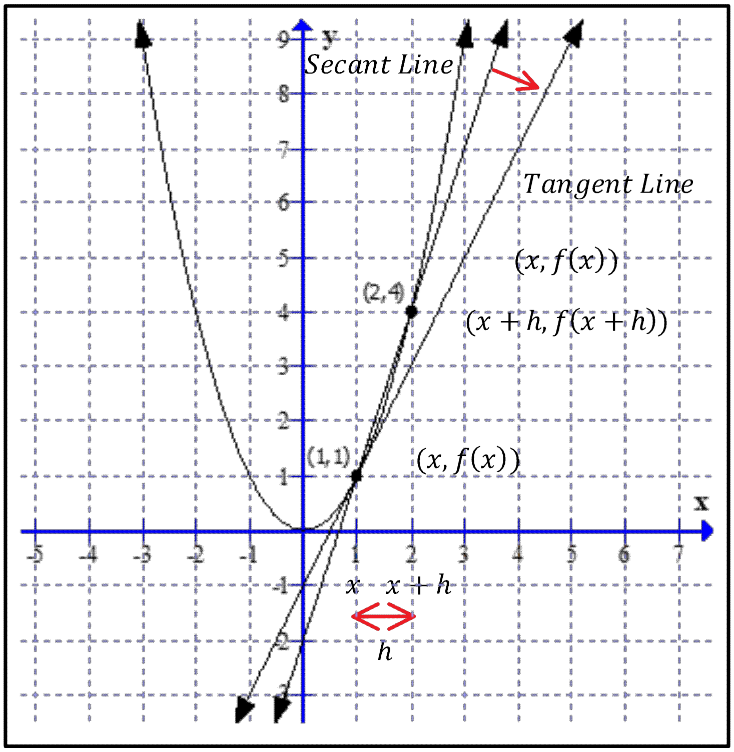
Find equation of tangent line to x^2 at $x = 1$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ **Slope**

$m^* = \frac{f(x+h) - f(x)}{x + h - x}$

$m^* = \frac{f(x+h) - f(x)}{h}$

x	y
2	4
1.01	1.0201
1.1	1.21
1	1



Find slope over domain

$y = x^2$ [1,2] [1,1.1] [1,1.01]

$h = 1$ $h = 0.1$ $h = 0.01$

(1,1) (2,4) (1.1,1.21) (1.01,1.0201)

$m = \frac{4-1}{2-1}$, $m = 3$

$m = \frac{1.21 - 1}{1.1 - 1}$, $m = 2.1$

$m = \frac{1.0201 - 1}{1.01 - 1}$, $m = 2.01$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Definition of the Derivative

$f(x) = x^2$

$f(x+h) = (x+h)^2$

$\lim_{h \to 0} \frac{(x+h)^2 - (x^2)}{h}$

$\lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$ — Foil, Simplify

$\lim_{h \to 0} \frac{2xh + h^2}{h}$ — Factor, Simplify

$\lim_{h \to 0} \frac{h(2x + h)}{h}$ — Substitute

$\lim_{h \to 0} 2x + h$

$f'(x) = 2x$

Slope of Tangent

$m = f'(1) = 2(1)$ $x = 1$

$m = f'(1) = 2$

Power Rule

$y = x^2$

$y' = 2x$

$m = 2(1)$

$m = 2$

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x - 1)$

$y = 2x - 2 + 1$

$y = 2x - 1$

$0 = 2x - y - 1$

Equation of Tangent Line

$y = x^2$

$y = (1)^2$

$y = 1$

(1,1)

$$m = f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Alternative methods.

$m = f'(1) = \lim_{x \to 1} \frac{x^2 - f(1)}{x - 1}$

$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

$\lim_{x \to 1} \frac{(x+1)(x-1)}{(x-1)}$

$\lim_{x \to 1} x + 1$

$1 + 1$

$m = f'(1) = 2$

$$m = f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$m = f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$

$\lim_{h \to 0} \frac{(1+h)^2 - (1)^2}{h}$

$\lim_{h \to 0} \frac{1 + 2h + h^2 - 1}{h}$

$\lim_{h \to 0} \frac{2h + h^2}{h}$

$\lim_{h \to 0} \frac{h(2+h)}{h}$

$\lim_{h \to 0} 2 + h$

$2 + 0$

$m = f'(1) = 2$