C12 - 2.4 - Definition of Derivative

$$y = x^n$$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\sum_{k=0}^{n}\left[\binom{n}{k} x^{n-k}h^k\right] - x^n\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[x^n + \binom{n}{1}x^{n-1}h + \binom{n}{2}x^{n-2}h^2 + \cdots + \binom{n}{n}h^n - x^n\right]$$

$$= \lim_{h\to 0}\left[\binom{n}{1}x^{n-1} + \cancel{\binom{n}{2}x^{n-2}h + \cdots + \binom{n}{n}h^{n-1}}\right] \quad 0$$

$$\frac{dy}{dx} = nx^{n-1} \quad QED$$

$$\boxed{n! = n(n-1)(n-2)\ldots\times 2\times 1}$$

$$\boxed{{}_nC_1 = \frac{n!}{1!\,(n-1)!} = \frac{n(n-1)!}{1(n-1)!} = n}$$

$$\boxed{\binom{n}{k} = {}_nC_k = \frac{n!}{k!\,(n-k)!}}$$

Newton's Binomial Theorem - 1729*

$$\boxed{(a+b)^n} = {}_nC_0(a)^n(b)^0 + {}_nC_1(a)^{n-1}(b)^1 + {}_nC_2(a)^{n-2}(b)^2 + \cdots + {}_nC_{n-1}(a)^1(b)^{n-1} + {}_nC_n(a)^0(b)^n$$

Pascal's Triangle - 1662*

$n = 0$	Row 1	1
$n = 1$	Row 2	1 1
$n = 2$	Row 3	1 2 1
$n = 3$	Row 4	1 3 3 1
$n = 4$	Row 5	1 4 6 4 1
...	...	...

$k = 0$

$k = 1$ ${}_4C_3$

Add #'s above*

$$\begin{array}{cc} 1 & 1 \\ \searrow & \swarrow \\ \multicolumn{2}{c}{2} \end{array}$$

$$\begin{array}{ccccc}
 & & {}_0C_0 & & \\
 & {}_1C_0 & & {}_1C_1 & \\
{}_2C_0 & & {}_2C_1 & & {}_2C_2 \\
\end{array}$$

$$\begin{array}{cccc} {}_3C_0 & {}_3C_1 & {}_3C_2 & {}_3C_3 \end{array}$$

$$\begin{array}{ccccc} {}_4C_0 & {}_4C_1 & {}_4C_2 & {}_4C_3 & {}_4C_4 \end{array}$$

$$(a+b)^0 = 1$$
$$(a+b)^1 = 1a + 1b$$
$$(a+b)^2 = 1a^2 + 2ab + 1b^1$$
$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$$
$$(a+b)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$$

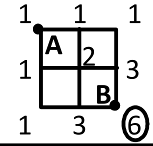
Arranging two of the Letters of ABCD.

AB BC CD
AC BD
AD

Logical Order.

$${}_4C_2 = \frac{4!}{2!\,(4-2)!} = \frac{4\times 3\times 2\times 1}{2\times 1\times (2)!} = 6$$

Pathways A→B (Down & Right)



2R'sD, 2D's

RRDD DDRR
RDRD DRDR
RDDR DRRD

How many words* from these letters.

PEEP

PEEP EPPE
PEPE EPEP
PPEE EEPP

$$\frac{4!}{2!\,2!} = 6$$

Gauss - 1855* Add #'s 1-50

$$\begin{array}{r} 1 + \ \ 2 + \cdots + 49 + \ 50 \\ + \ 50 + 49 + \cdots + \ \ 2 + \ \ 1 \\ \hline 51 + 51 + \cdots + \ 51 + 51 \end{array}$$

$$= \frac{51\times 50}{2} \qquad \text{Factor } \frac{50}{2}$$

$$s_{50} = \frac{50}{2}(1+50) \qquad \boxed{s_n = \frac{n}{2}(t_1 + t_n)}$$

$$s_{50} = 1275 \qquad 51 = 1 + 50$$