

# C12 - 2.4 - Definition of Derivative Equation Graph Notes

Find equation of tangent line to  $x^2$  at  $x = 1$ .

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \boxed{\text{Slope}}$$

$$m^* = \frac{f(x+h) - f(x)}{x+h-x}$$

$$m^* = \frac{f(x+h) - f(x)}{h}$$

$x$	$y$
2	4
1.01	1.0201
1.1	1.21
1	1

Find slope over domain

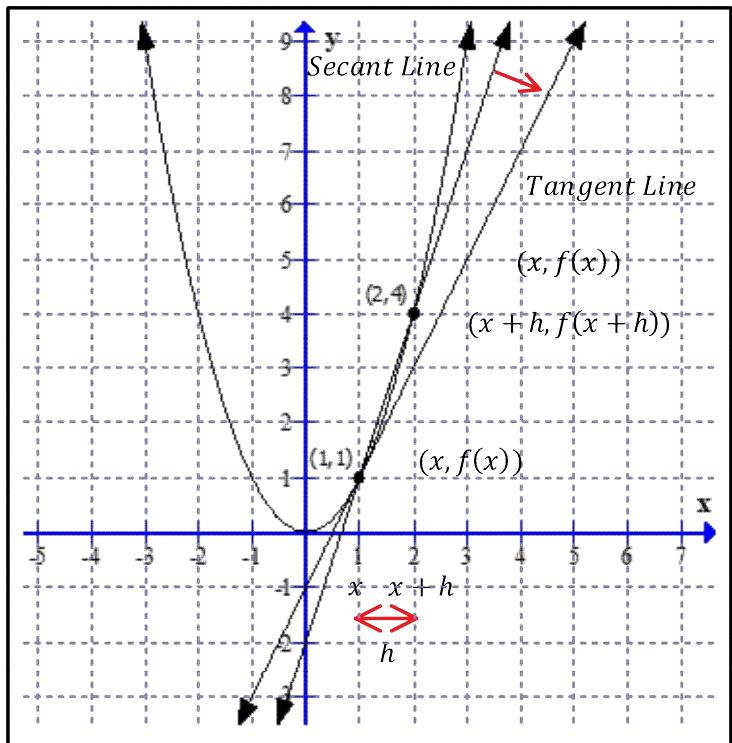
$$y = x^2 \quad [1, 2] \quad [1, 1.1] \quad [1, 1.01]$$

$$h = 1 \quad h = 0.1 \quad h = 0.01$$

(1,1) (2,4) (1.1,1.21) (1.01,1.0201)

$$m = \frac{4-1}{2-1} \quad m = \frac{1.21-1}{1.1-1} \quad m = \frac{1.0201-1}{1.01-1}$$

$$\boxed{m=3} \quad \boxed{m=2.1} \quad \boxed{m=2.01}$$



$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{(x+h)^2 - (x^2)}{h} \leftarrow \\ & \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ & \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} \\ & \lim_{h \rightarrow 0} \frac{h(2x+h)}{h} \\ & \lim_{h \rightarrow 0} 2x+h \end{aligned}$$

$$f'(x) = \boxed{2x}$$

Slope of Tangent

$$m = f'(1) = \frac{2(1)}{2} \quad x = 1$$

$$m = f'(1) = \boxed{2}$$

$$y - y_1 = m(x - x_1)$$

$$\boxed{y - 1 = 2(x - 1)}$$

$$y = 2x - 2 + 1$$

$$\boxed{y = 2x - 1}$$

$$0 = 2x - y - 1$$

Equation of Tangent Line

Power Rule

$$y = x^2$$

$$y' = 2x$$

$$m = 2(1)$$

$$\boxed{m = 2}$$

$$y = x^2$$

$$y = (1)^2$$

$$y = 1$$

$$\boxed{(1,1)}$$

Definition of the Derivative

$$f(x) = x^2$$

$$f(x+h) = (x+h)^2$$

Foil  
Simplify  
Factor, Simplify  
Substitute

$$m = f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$m = f'(1) = \lim_{x \rightarrow 1} \frac{x^2 - f(1)}{x - 1}$$

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$$

Alternative methods.

$$\lim_{x \rightarrow 1} \frac{(x+1)(x-1)}{(x-1)}$$

$$\lim_{x \rightarrow 1} \frac{x+1}{1+1}$$

$$\boxed{m = f'(1) = 2}$$

$$m = f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$m = f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$$

$$\lim_{h \rightarrow 0} \frac{(1+h)^2 - (1)^2}{h}$$

$$\lim_{h \rightarrow 0} \frac{1+2h+h^2 - 1}{h}$$

$$\lim_{h \rightarrow 0} \frac{2h+h^2}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(2+h)}{h}$$

$$\lim_{h \rightarrow 0} \frac{2+h}{2+0}$$

$$\boxed{m = f'(1) = 2}$$

## C12 - 2.4 - Definition of Derivative

$$\begin{aligned}
 y &= x^n \\
 \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{(x+h)^n - x^n}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \sum_{k=0}^n \left[ \binom{n}{k} x^{n-k} h^k \right] - x^n \right] \\
 &= \lim_{h \rightarrow 0} \frac{1}{h} [x^n + \binom{n}{1} x^{n-1} h + \binom{n}{2} x^{n-2} h^2 + \cdots + \binom{n}{n} h^n - x^n] \\
 &= \lim_{h \rightarrow 0} [\binom{n}{1} x^{n-1} + \binom{n}{2} x^{n-2} h + \cdots + \binom{n}{n} h^{n-1}] \\
 \frac{dy}{dx} &= nx^{n-1} \quad QED
 \end{aligned}$$

$n! = n(n-1)(n-2) \dots \times 2 \times 1$

$nC_1 = \frac{n!}{1!(n-1)!}$   
 $= \frac{n(n-1)!}{1(n-1)!} = \boxed{n}$

$\binom{n}{k} = {}_n C_k = \frac{n!}{k!(n-k)!}$

Newton's Binomial Theorem - 1729\*

$$(a+b)^n = {}_n C_0 (a)^n (b)^0 + {}_n C_1 (a)^{n-1} (b)^1 + {}_n C_2 (a)^{n-2} (b)^2 + \cdots + {}_n C_{n-1} (a)^1 (b)^{n-1} + {}_n C_n (a)^0 (b)^n$$

Pascal's Triangle - 1662\*

$$\begin{array}{ll}
 n=0 \text{ Row 1} & 1 \\
 n=1 \text{ Row 2} & 1 \ 1 \\
 n=2 \text{ Row 3} & 1 \ 2 \ 1 \\
 n=3 \text{ Row 4} & 1 \ 3 \ 3 \ 1 \\
 n=4 \text{ Row 5} & 1 \ 4 \ 6 \ 4 \ 1 \\
 \dots & \dots
 \end{array}$$

Add #'s above\*  
  
 $k=0$     $k=1$     ${}_4 C_3$

$$\begin{array}{ccccccc}
 & & {}_0 C_0 & & & & \\
 & & {}_1 C_0 & {}_1 C_1 & & & \\
 & & {}_2 C_0 & {}_2 C_1 & {}_2 C_2 & & \\
 & & {}_3 C_0 & {}_3 C_1 & {}_3 C_2 & {}_3 C_3 & \\
 {}_4 C_0 & {}_4 C_1 & {}_4 C_2 & {}_4 C_3 & {}_4 C_4 & &
 \end{array}$$

$$\begin{aligned}
 (a+b)^0 &= 1 \\
 (a+b)^1 &= 1a + 1b \\
 (a+b)^2 &= 1a^2 + 2ab + 1b^2 \\
 (a+b)^3 &= 1a^3 + 3a^2b + 3ab^2 + 1b^3 \\
 (a+b)^4 &= 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4
 \end{aligned}$$

Arranging two of the Letters of ABCD.

$$\begin{array}{l}
 \text{AB BC CD} \\
 \text{AC BD} \\
 \text{AD}
 \end{array}$$

Logical Order.  
4!

$$\begin{aligned}
 {}_4 C_2 &= \frac{2!(4-2)!}{4 \times 3 \times 2 \times 1} \\
 &= \frac{2 \times 1 \times (2)!}{2 \times 1 \times (2)!} \\
 &= \boxed{6}
 \end{aligned}$$

Pathways A→B  
(Down & Right)

1	1	1
A	2	3
1	B	6
1	3	

2R'sD, 2D's  
 RRDD DRRR  
 RDRD DRDR  
 RDDR DRRD

How many words\* from these letters.

PEEP	EPPE
PEPE	EPEP
PPEE	EEPP
PEEP	$\frac{4!}{2!2!} = \boxed{6}$

Gauss - 1855\* Add #'s 1-50

$$\begin{aligned}
 &1 + 2 + \cdots + 49 + 50 \\
 &+ 50 + 49 + \cdots + 2 + 1 \\
 &\hline
 &51 + 51 + \cdots + 51 + 51 \\
 &= \frac{51 \times 50}{2} \quad \text{Factor } \frac{50}{2}
 \end{aligned}$$

$$\begin{aligned}
 s_{50} &= \frac{50}{2}(1+50) \\
 s_{50} &= \boxed{1275} \quad s_n = \frac{n}{2}(t_1 + t_n) \\
 &\quad 51 = 1 + 50
 \end{aligned}$$

Don't forget to write the formula!

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

## C12 - 2.4 - Definition of Derivative

$$f(x) = x^2$$

$$\lim_{h \rightarrow 0} \frac{(x+h)^2 - (x^2)}{h}$$

$$\lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$\lim_{h \rightarrow 0} \frac{2xh + h^2}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(2x+h)}{h}$$

$$\lim_{h \rightarrow 0} \frac{2x+h}{h}$$

$$f'(x) = \textcircled{2x}$$

$$f(x) = x^3$$

$$\lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$$

$$\lim_{h \rightarrow 0} \frac{(x+h)(x+h)(x+h) - x^3}{h}$$

$$\lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h}$$

$$f'(x) = \textcircled{3x^2}$$

$$f(x) = x^4$$

$$\lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h}$$

$$\lim_{h \rightarrow 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(4x^3 + 6x^2h + 4xh^2 + h^3)}{h}$$

$$f'(x) = \textcircled{4x^3}$$

Diff of Squares

$$(x+h)^2 - x^2$$

$$((x+h)+x)((x+h)-x)$$

$$2xh + h^2$$

$$\dots$$

Diff of Cubes/Binomial Theorem

$$(x+h)^3 - x^3$$

$$((x+h)-x)((x+h)^2 + x(x+h) + x^2)$$

$$\dots$$

$$(x+h)^3 - x^3$$

$$x^3 + x^2h + xh^2 + h^3 - x^3$$

$$\dots$$

FOIL/Diff of Squares

$$(x+h)^4 - x^2$$

$$(x+h)(x+h)(x+h)(x+h) - x^2$$

$$\dots$$

$$(x+h)^4 - x^2$$

$$((x+h)^2 - x)((x+h)^2 + x)$$

$$\dots$$

$$f(x) = x^2 - x$$

$$\lim_{h \rightarrow 0} \frac{(x+h)^2 - (x+h) - (x^2 - x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h}$$

$$\lim_{h \rightarrow 0} \frac{2xh + h^2 - h}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(2x + h - 1)}{h}$$

$$f'(x) = \textcircled{2x - 1}$$

$$f(x) = \frac{1}{x}$$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \times LCD$$

$$\lim_{h \rightarrow 0} \frac{x - (x+h)}{hx(x+h)} \times LCD$$

$$\lim_{h \rightarrow 0} \frac{-h}{hx(x+h)} \times LCD$$

$$\lim_{h \rightarrow 0} \frac{-1}{x(x+h)} \times LCD$$

$$f'(x) = \textcircled{-\frac{1}{x^2}}$$

$$f(x) = \frac{1}{x^2}$$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \times LCD$$

$$\lim_{h \rightarrow 0} \frac{x^2 - (x+h)^2}{hx^2(x+h)^2} \times LCD$$

$$\lim_{h \rightarrow 0} \frac{hx^2(x+h)^2}{x^2 - x^2 - 2xh - h^2} \times LCD$$

$$\lim_{h \rightarrow 0} \frac{h(-2x-h)}{hx^2(x+h)^2} \times LCD$$

$$\lim_{h \rightarrow 0} \frac{-2x-h}{x^2(x+h)^2} \times LCD$$

$$\frac{-2x}{x^4} \times LCD$$

$$f'(x) = \textcircled{-\frac{2}{x^3}}$$

$$f(x) = \sqrt{x}$$

$$f(x) = \frac{1}{\sqrt{x}}$$

$$\lim_{h \rightarrow 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} \times Conj$$

$$\lim_{h \rightarrow 0} \frac{h}{\sqrt{x+h} - \sqrt{x}} \times Conj$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \times Conj$$

$$\lim_{h \rightarrow 0} \frac{x - (x+h)}{h\sqrt{x}\sqrt{x+h}} \times Conj$$

$$\lim_{h \rightarrow 0} \frac{-h}{h\sqrt{x}\sqrt{x+h} + h(\sqrt{x} + \sqrt{x+h})} \times Conj$$

$$\lim_{h \rightarrow 0} \frac{-1}{h(\sqrt{x}\sqrt{x} + (\sqrt{x} + \sqrt{x+h}))} \times Conj$$

$$\frac{-1}{\sqrt{x}\sqrt{x} + (\sqrt{x} + \sqrt{x+h})} \times Conj$$

$$\frac{-1}{\sqrt{x}\sqrt{x} + (\sqrt{x} + \sqrt{x})} \times Conj$$

$$f'(x) = \textcircled{-\frac{1}{2x\sqrt{x}}}$$

OR Add Fractions, Flip and Multiply

$$\frac{1}{x+h} - \frac{1}{x} = \frac{1}{(x+h)^2} - \frac{1}{x^2}$$

$$\frac{h}{x - (x+h)} \times \frac{1}{h} = \frac{h}{x^2 - (x+h)^2} \times \frac{1}{h}$$

$$\frac{h}{x(x+h)} \times \frac{1}{h} = \frac{1}{x^2(x+h)^2} \times \frac{1}{h}$$

Don't forget to write the formula!

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

## C12 - 2.4 - Definition of Derivative

$$f(x) = \frac{3x}{x+1}$$

$$\begin{aligned} & \underset{h \rightarrow 0}{\text{Lim}} \frac{\frac{3(x+h)}{x+h+1} - \frac{3x}{x+1}}{h} \times \text{LCD} \\ & \underset{h \rightarrow 0}{\text{Lim}} \frac{3(x+h)(x+1) - 3x(x+h+1)}{h(x+1)(x+h+1)} \times \text{LCD} \\ & \underset{h \rightarrow 0}{\text{Lim}} \frac{3x^2 + 3x + 3hx + 3h - 3x^2 - 3xh - 3x}{h(x+1)(x+h+1)} \\ & \underset{h \rightarrow 0}{\text{Lim}} \frac{3h}{h(x+1)(x+h+1)} \\ & \underset{h \rightarrow 0}{\text{Lim}} \frac{3}{(x+1)(x+h+1)} \\ f'(x) &= \frac{3}{(x+1)^2} \end{aligned}$$

$$f(x) = 2x^2 - 5x$$

$$\begin{aligned} & \underset{h \rightarrow 0}{\text{Lim}} \frac{2(x+h)^2 - 5(x+h) - (2x^2 - 5x)}{h} \\ & \underset{h \rightarrow 0}{\text{Lim}} \frac{2x^2 + 4xh + 2h^2 - 5x - 5h - 2x^2 + 5x}{h} \\ & \underset{h \rightarrow 0}{\text{Lim}} \frac{h(4x + 2h - 5)}{h} \\ & \underset{h \rightarrow 0}{\text{Lim}} 4x + 2h - 5 \\ f'(x) &= \boxed{4x - 5} \end{aligned}$$

$$f(x) = 2x - 1$$

$$\begin{aligned} & \underset{h \rightarrow 0}{\text{Lim}} \frac{2(x+h) - 1 - (2x - 1)}{h} \\ & \underset{h \rightarrow 0}{\text{Lim}} \frac{2x + 2h - 1 - 2x + 1}{h} \\ & \frac{2h}{h} \\ f'(x) &= \boxed{2} \quad y = mx + b \end{aligned}$$

$$f(x) = \sin x$$

$$\begin{aligned} & \underset{h \rightarrow 0}{\text{Lim}} \frac{\sin(x+h) - \sin x}{h} \quad \sin(a+b) = \sin a \cos b + \sin b \cos a \\ & \underset{h \rightarrow 0}{\text{Lim}} \frac{\sin x \cosh + \sinh \cos x - \sin x}{h} \\ & \underset{h \rightarrow 0}{\text{Lim}} \frac{\sin x \cosh - \sin x}{h} + \frac{\sinh \cos x}{h} \\ & \underset{h \rightarrow 0}{\text{Lim}} \frac{\sin x \frac{(\cosh - 1)}{h} + \frac{\sinh}{h} \cosh}{\sin x \times 0 + 1 \times \cos x} \\ f'(x) &= \boxed{\cos x} \end{aligned}$$

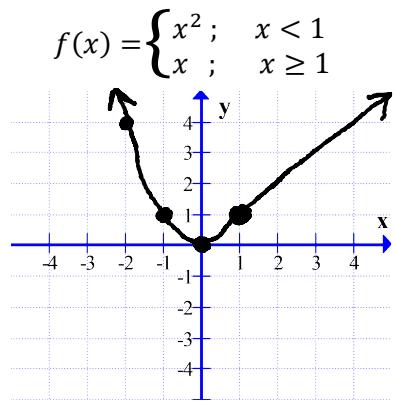
$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$$

$$\begin{aligned} f(x) &= x \\ & \underset{h \rightarrow 0}{\text{Lim}} \frac{(x+h) - (x)}{h} \\ & \frac{h}{h} \\ h &\rightarrow 0 \end{aligned}$$

$$f'(x) = \boxed{1} \quad y = mx + b$$

## C12 - 2.4 - Piece Derivatives Notes



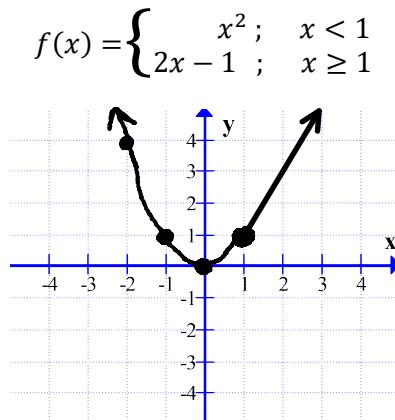
$$f'(x) = \lim_{h \rightarrow 0^-} \frac{f(x+h) - f(x)}{h} = 2$$

$\neq$  Not Differentiable

$$f'(x) = \lim_{h \rightarrow 0^+} \frac{f(x+h) - f(x)}{h} = 1$$

$$\begin{aligned} f(x) &= x^2 & f(x) &= x \\ \lim_{h \rightarrow 0^-} \frac{f(x+h) - f(x)}{h} &= \lim_{h \rightarrow 0^+} \frac{f(x+h) - f(x)}{h} & \lim_{h \rightarrow 0^+} \frac{(x+h) - (x)}{h} &= \\ \lim_{h \rightarrow 0^-} \frac{(x+h)^2 - (x^2)}{h} &= \lim_{h \rightarrow 0^+} \frac{h}{h} & \lim_{h \rightarrow 0^+} \frac{x^2 + 2xh + h^2 - x^2}{h} &= y = mx + b \\ \lim_{h \rightarrow 0^-} \frac{2xh + h^2}{h} &= \lim_{h \rightarrow 0^+} \frac{h}{h} & f(x) &= x \\ \lim_{h \rightarrow 0^-} \frac{h(2x + h)}{h} &= \lim_{h \rightarrow 0^+} \frac{h}{h} & f'(x) &= 1 \\ \lim_{h \rightarrow 0^-} 2x + h &= \lim_{h \rightarrow 0^+} 1 & f'(1) &= 1^* \end{aligned}$$

$$\begin{aligned} f'(x) &= 2x & f(x) &= x^2 \\ m = f'(1) &= 2(1) & f'(x) &= 2x \\ m = f'(1) &= 2 & f'(1) &= 2(1) \\ m &= f'(1) = 2 & f'(2) &= 2^* \end{aligned}$$



$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0^-} \frac{f(x+h) - f(x)}{h} = 2 & \text{Differentiable} \\ f'(x) &= \lim_{h \rightarrow 0^+} \frac{f(x+h) - f(x)}{h} = 2 \end{aligned}$$

$$\begin{aligned} f(x) &= 2x - 1 & f(x) &= mx + b \\ \lim_{h \rightarrow 0^+} \frac{f(x+h) - f(x)}{h} &= \lim_{h \rightarrow 0^+} \frac{2(x+h) - 1 - (2x - 1)}{h} & f(x) &= 2x - 1 \\ \lim_{h \rightarrow 0^+} \frac{2x + 2h - 1 - 2x + 1}{h} &= \lim_{h \rightarrow 0^+} \frac{2h}{h} & f'(x) &= 2 \\ \lim_{h \rightarrow 0^+} \frac{2h}{h} &= \lim_{h \rightarrow 0^+} 2 & f'(1) &= 2^* \\ f'(x) &= 2 \end{aligned}$$

Find k so differentiable.

$$f(x) = \begin{cases} 2x - k & x \geq 1 \\ x^2 & x < 1 \end{cases}$$

$$2x - k = x^2$$

$$2(1) - k = (1)^2$$

$$2 - k = 1$$

$$k = 1$$

Sub x value  
and equate

$$\begin{aligned} \text{Take derivatives} \\ \text{Sub x and equate} \\ f(x) &= 2x - k & f(x) &= x^2 \\ f'(x) &= 2 & f'(x) &= 2x \\ f'(2) &= 2 & f'(1) &= 2(1) \\ 2 &= 2 & f'(1) &= 2 \end{aligned}$$

Nothing to solve for.