C12 - 2.5 - Definition of Derivative

Don't forget to write the formula!

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$f(x) = x^2$

$$\lim_{h\to 0} \frac{(x+h)^2-(x^2)}{h}$$
$$\lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h}$$
$$\lim_{h\to 0} \frac{2xh+h^2}{h}$$ FOIL
$$\lim_{h\to 0} \frac{h(2x+h)}{h}$$
$$\lim_{h\to 0} 2x+h$$
$$f'(x) = 2x$$

$f(x) = x^3$

$$\lim_{h\to 0} \frac{(x+h)^3-x^3}{h}$$
$$\lim_{h\to 0} \frac{(x+h)(x+h)(x+h)-x^3}{h}$$
$$\lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$
$$\lim_{h\to 0} \frac{h(3x^2+3xh+h^2)}{h}$$
$$f'(x) = 3x^2$$

$f(x) = x^4$

$$\lim_{h\to 0} \frac{(x+h)^4-x^4}{h}$$ Binomial Theorem
$$\lim_{h\to 0} \frac{x^4+4x^3h+6x^2h^2+4xh^3+h^4-x^3}{h}$$
$$\lim_{h\to 0} \frac{h(4x^3+6x^2h+4xh^2+h^3)}{h}$$
$$f'(x) = 4x^3$$

Diff of Squares

$(x+h)^2-x^2$

$((x+h)+x)((x+h)-x)$

$2xh+h^2$

…

Diff of Cubes/Binomial Theorem

$(x+h)^3-x^3$

$((x+h)-x)((x+h)^2+x(x+h)+x^2)$

…

$(x+h)^3-x^3$

$1x^3+3x^2h+3xh^2+1h^3-x^3$

…

FOIL/Diff of Squares

$(x+h)^4-x^2$

$(x+h)(x+h)(x+h)(x+h)-x^2$

…

$(x+h)^4-x^2$

$((x+h)^2-x)((x+h)^2+x)$

…

$f(x) = x^2 - x$

$$\lim_{h\to 0} \frac{(x+h)^2-(x+h)-(x^2-x)}{h}$$
$$\lim_{h\to 0} \frac{x^2+2xh+h^2-x-h-x^2+x}{h}$$
$$\lim_{h\to 0} \frac{2xh+h^2-h}{h}$$
$$\lim_{h\to 0} \frac{h(2x+h-1)}{h}$$
$$f'(x) = 2x-1$$

$f(x) = \frac{1}{x}$

$$\lim_{h\to 0} \frac{\frac{1}{x+h}-\frac{1}{x}}{h} \quad \frac{\times LCD}{\times LCD}$$
$$\lim_{h\to 0} \frac{x-(x+h)}{hx(x+h)}$$
$$\lim_{h\to 0} \frac{-h}{hx(x+h)}$$
$$\lim_{h\to 0} \frac{-1}{x(x+h)}$$
$$f'(x) = -\frac{1}{x^2}$$

$f(x) = \frac{1}{x^2}$

$$\lim_{h\to 0} \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} \quad \frac{\times LCD}{\times LCD}$$
$$\lim_{h\to 0} \frac{x^2-(x+h)^2}{hx^2(x+h)^2}$$
$$\lim_{h\to 0} \frac{x^2-x^2-2xh-h^2}{hx^2(x+h)^2}$$
$$\lim_{h\to 0} \frac{h(-2x-h)}{hx^2(x+h)^2}$$
$$\lim_{h\to 0} \frac{-2x-h}{x^2(x+h)^2}$$
$$\frac{-2x}{x^4}$$
$$f'(x) = -\frac{2}{x^3}$$

$f(x) = \sqrt{x}$

$$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \quad \frac{\times Conj}{\times Conj}$$
$$\lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}-\sqrt{x})}$$
$$\lim_{h\to 0} \frac{h}{h(\sqrt{x+h}-\sqrt{x})}$$
$$\lim_{h\to 0} \frac{1}{(\sqrt{x+h}-\sqrt{x})}$$
$$f'(x) = \frac{1}{2\sqrt{x}}$$

$f(x) = \frac{1}{\sqrt{x}}$

$$\lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h} \quad \frac{\times LCD}{\times LCD}$$
$$\lim_{h\to 0} \frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}} \quad \frac{\times Conj}{\times Conj}$$
$$\lim_{h\to 0} \frac{x-(x+h)}{h\sqrt{x}\sqrt{x}+h(\sqrt{x}+\sqrt{x+h})}$$
$$\lim_{h\to 0} \frac{-h}{h(\sqrt{x}\sqrt{x}+(\sqrt{x}+\sqrt{x+h}))}$$
$$\frac{-1}{\sqrt{x}\sqrt{x}+(\sqrt{x}+\sqrt{x+h})}$$
$$\frac{-1}{\sqrt{x}\sqrt{x}+(\sqrt{x}+\sqrt{x})}$$
$$f'(x) = -\frac{1}{2x\sqrt{x}} = -\frac{1}{2x^{\frac{3}{2}}}$$

OR Add Fractions, Flip and Multiply

$$\frac{\frac{1}{x+h}-\frac{1}{x}}{h} \qquad \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$
$$\frac{x-(x+h)}{x(x+h)} \times \frac{1}{h} \qquad \frac{x^2-(x+h)^2}{x^2(x+h)^2} \times \frac{1}{h}$$

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$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f(x) = \frac{3x}{x+1}$$

$$\lim_{h\to 0} \frac{\frac{3(x+h)}{x+h+1} - \frac{3x}{x+1}}{h} \quad \frac{\times LCD}{\times LCD}$$

$$\lim_{h\to 0} \frac{3(x+h)(x+1) - 3x(x+h+1)}{h(x+1)(x+h+1)}$$

$$\lim_{h\to 0} \frac{3x^2+3x+3hx+3h-3x^2-3xh-3x}{h(x+1)(x+h+1)}$$

$$\lim_{h\to 0} \frac{3h}{h(x+1)(x+h+1)}$$

$$\lim_{h\to 0} \frac{3}{(x+1)(x+h+1)}$$

$$f'(x) = \frac{3}{(x+1)^2}$$

$$f(x) = 2x^2 - 5x$$

$$\lim_{h\to 0} \frac{2(x+h)^2 - 5(x+h) - (2x^2-5x)}{h}$$

$$\lim_{h\to 0} \frac{2x^2+4xh+2h^2-5x-5h-2x^2+5x}{h}$$

$$\lim_{h\to 0} \frac{h(4x+2h-5)}{h}$$

$$\lim_{h\to 0} 4x+2h-5$$

$$f'(x) = 4x-5$$

$$f(x) = 2x-1$$

$$\lim_{h\to 0} \frac{2(x+h)-1-(2x-1)}{h}$$

$$\lim_{h\to 0} \frac{2x+2h-1-2x+1}{h}$$

$$\frac{2h}{h}$$

$$f'(x) = 2 \qquad y = mx+b$$

$$f(x) = x$$

$$\lim_{h\to 0} \frac{(x+h)-(x)}{h}$$

$$\lim_{h\to 0} \frac{h}{h}$$

$$f'(x) = 1 \qquad y = mx+b$$

$$f(x) = sinx$$

$$\lim_{h\to 0} \frac{sin(x+h) - sinx}{h} \qquad \sin(a+b) = sinacosb + sinbcosa$$

$$\lim_{h\to 0} \frac{sinxcosh + sinhcosx - sinx}{h}$$

$$\lim_{h\to 0} \frac{sinxcosh - sinx}{h} + \frac{sinhcosx}{h}$$

$$\lim_{h\to 0} sinx\frac{(cosh-1)}{h} + \frac{sinh}{h}\ cosh$$

$$sinx \times 0 + 1 \times cosx$$

$$f'(x) = cosx$$

$$\lim_{x\to 0} \frac{sinx}{x} = 1$$

$$\lim_{x\to 0} \frac{cosx-1}{x} = 0$$