C12 - 2.8 - Chain Rule Derivatives Notes

$y = (2x+1)^3$ Chain Rule
$y' = 3(2x+1)^{3-1} \times 2$
$y' = 6(2x+1)^2$

Inside
$y = 2x+1$
$y' = 2$

$y = x^3$ Chain Rule
$\frac{dy}{dx} = 3x^2 \times \frac{dx}{dx}$
$\frac{dy}{dx} = 3x^2 \times 1$

$y = (2x+1)^3$ FOIL
$y = 8x^3 + 12x^2 + 6x + 1$
$y' = 24x^2 + 24x + 6$
$y' = 6(4x^2 + 4x + 1)$
$y' = 6(2x+1)^2$

$y = (x^2+2x)^5$
$y' = 5(x^2+2x)^4(2x+2)$
$y' = 10(x^2+2x)^4(x+1)$ GCF

$y = x^2 + 2x$
$y' = 2x + 2$

$y = that^{\#}$
$y' = \#that^{\#-1} \times (chain\ that)$

$y = \sqrt{3x}$
$y = (3x)^{\frac{1}{2}}$
$y' = \frac{1}{2}(3x)^{-\frac{1}{2}} \times 3$
$y' = \frac{3}{2(3x)^{\frac{1}{2}}}$
$y' = \frac{3}{2\sqrt{3x}}$

$y = 3x$
$y' = 3$

$y = \sqrt{that}$
$y' = \frac{chain\ that}{2\sqrt{that}}$

$y = u^3$
$y' = 3u^2(u')$
$y' = 3(2x+1)^2(2)$
$y' = 6(2x+1)^2$

$u = 2x+1$
$u' = 2$

$y = (2x+1)^3$
...

$y = u^4 + 5u^2$
$\frac{dy}{dx} = 4u^3(u') + 10u(u')$
$\frac{dy}{dx} = 4(x^5+2x^2+1)^3(5x^4+4x) + 10(x^5+2x^2+4)(5x^4+4x)$
$y = (x^5+2x^2+1)^4 + 5(x^5+2x^2+1)^2$
...

$u = x^5 + 2x^2 + 1$
$u' = 5x^4 + 4x$

$y = x^3(2x-5)^4$
$y' = 3x^2(2x-5)^4 + 4(2x-5)^3(2)(x^3)$
$y' = 3x^2(2x-5)^4 + 8x^3(2x-5)^3)$
$y' = x^2(2x-5)^3(3(2x-5) + 8x)$
$y' = x^2(2x-5)^3(14x-5)$

$u = x^3$
$u' = 3x^2$

$v = (2x-5)^4$
$v' = 4(2x-5)^3(2)$

2D/3D

$A = \pi r^2$
$C = \frac{DA}{dr} = 2\pi r \times \frac{dr}{dr}$

$V = \frac{4}{3}\pi r^3$
$SA = \frac{DV}{r} = 4\pi r^2 \times \frac{dr}{dr}$

$y = \frac{x}{\sqrt{2x+3}}$

$u = x$
$u' = 1$

$v = \sqrt{2x+3}$
$v' = \frac{2}{\sqrt{2x+3}}$

$y' = \frac{1(\sqrt{2x+3}) + \frac{2}{\sqrt{2x+3}}}{2x+3}$ $\times LCD$ $\times LCD$

$y' = \frac{2x+3+2}{(2x+3)^{\frac{3}{2}}} = \frac{2x+5}{(2x+3)^{\frac{3}{2}}}$

$y = \left(\frac{x+5}{2x-1}\right)^3$
$y' = 3\left(\frac{x+5}{2x-1}\right)^2\left(\frac{1(2x-1) - 2(x+5)}{(2x-1)^2}\right)$
$y' = 3\left(\frac{x+5}{2x-1}\right)^2\left(\frac{-8}{(2x-1)^2}\right)$
$y' = \frac{-24(x+5)^2}{(2x-1)^4}$

$y = (2x + (x^2+1)^2)^3$
$y' = 3(2x + (x^2+1)^2)^2(2 + 2(x^2+1)^1(2x))$
$y' = 3(2x + (x^2+1)^2)^2(4x^3 + 4x + 2)$

Inside
$y = 2x + (x^2+1)^2$
$y' = 2 + 2(x^2+1)^1(2x)$

$\frac{d}{dx}x^3 = 3x^2 \times \frac{dx}{dx}$
$\frac{d}{dx}y^3 = 3y^2 \times \frac{dy}{dx}$
$\frac{d}{dy}x^3 = 3x^2 \times \frac{dx}{dy}$
$\frac{d}{dy}y^3 = 3y^2 \times \frac{dy}{dy}$

$y' = \frac{dy}{dx}$