

C12 - 2.9 - Ln Derivatives Notes

$$y' = \frac{dy}{dx} = f'(x)$$

Log Rules

$$\begin{aligned} y &= \ln x & y &= \log_5 x & y &= \ln 2x & y &= \log_5 2x & y &= \log_7 x^2 & y &= \log_7 x^2 & y &= \log_5 \sqrt{x} \\ y' &= \frac{1}{x \ln e} & y' &= \frac{1}{x \ln 5} & y' &= \frac{1}{x} \times 2 & y' &= \frac{1}{2x} \frac{1}{\ln 5} (2) & y' &= \frac{1}{x^2 \ln 7} \times 2x & y &= 2 \log_7 x & y &= \log_5 x^{\frac{1}{2}} \\ y' &= \frac{1}{x} & & & y' &= \frac{2}{x} & y' &= \frac{1}{x \ln 5} & y' &= 2 \times \frac{1}{x \ln 7} & y &= \frac{1}{2} \log_5 x & y' &= \frac{1}{2} \left(\frac{1}{x \ln 5} \right) \end{aligned}$$

$$\begin{aligned} y &= \ln(\text{that}) \\ y' &= \frac{\text{chain that}}{\text{that}} \end{aligned}$$

$$\begin{aligned} y &= \log_b(\text{that}) \\ y' &= \frac{\text{chain that}}{\text{that}(ln b)} \end{aligned}$$

$$\begin{aligned} y &= \ln(lnx) & y &= \ln(x^2) & y &= \ln(x^2) & y &= \ln(1 + x^2) \\ y' &= \frac{1}{lnx} \times \frac{1}{x} & y' &= \frac{1}{x^2} \times 2x & y &= 2lnx & y' &= \frac{1}{1 + x^2} \times 2x \\ y' &= \frac{1}{xlnx} & y' &= \frac{2}{x} & y' &= 2 \times \frac{1}{x} & y' &= \frac{2x}{1 + x^2} \end{aligned}$$

$$\begin{aligned} y &= \log_2(3x + 1) \\ y' &= \frac{1}{(3x + 1) \ln 2} (3) \\ y' &= \frac{3}{(3x + 1) \ln 2} \end{aligned}$$

$$\begin{aligned} y &= (lnx)^2 \\ y' &= 2(lnx)^1 \times \frac{1}{x} \\ y' &= \frac{2lnx}{x} \end{aligned}$$

$$\begin{aligned} y &= \ln(x\sqrt{x-1}) \\ y &= lnx + ln\sqrt{x-1} \\ y' &= \frac{1}{x} + \frac{1}{\sqrt{x-1}} \left(\frac{1}{2\sqrt{x-1}} \right) \\ y' &= \frac{1}{x} + \frac{1}{2x-2} \\ y' &= \frac{3x-2}{x(2x-2)} \end{aligned}$$

$$\begin{aligned} y &= \ln\left(\frac{x+1}{x-1}\right) \\ y &= \ln(x+1) - \ln(x-1) \\ y' &= \frac{1}{x+1} - \frac{1}{x-1} \\ y' &= -\frac{z}{(x^2-1)} \end{aligned}$$

$$\begin{aligned} y &= xlnx \\ y' &= 1(lnx) + \frac{1}{x} \times x \\ y' &= lnx + 1 \end{aligned}$$

$$\begin{aligned} y &= lnx^x \\ y &= xlnx \\ y' &= lnx + 1 \end{aligned}$$

$$\begin{aligned} y &= lnxlnx \\ y' &= \frac{1}{x} lnx + \frac{1}{x} \times lnx \\ y' &= \frac{2lnx}{x} \end{aligned}$$

$$\begin{aligned} y &= x^x \\ lny &= lnx^x \quad \text{Ln Both Sides} \\ lny &= xlnx \\ \frac{1}{y} \times y' &= 1(lnx) + \frac{1}{x} \times x \\ \frac{y'}{y} &= lnx + 1 \\ y' &= y(lnx + 1) \\ y' &= x^x(lnx + 1) \end{aligned}$$

$$\begin{aligned} y &= x^{lnx} \\ lny &= lnx^{lnx} \\ lny &= lnxlnx \\ \frac{y'}{y} &= \frac{1}{x} lnx + \frac{1}{x} \times lnx \\ \frac{y'}{y} &= \frac{lnx}{x} + \frac{lnx}{x} \\ \frac{y'}{y} &= \frac{2lnx}{x} \\ y' &= y \left(\frac{2lnx}{x} \right) \\ y' &= x^{lnx} \left(\frac{2lnx}{x} \right) \end{aligned}$$

$$\begin{aligned} y &= \frac{(2x+1)^2}{(x+2)^3} \quad \text{A difficult quotient} \\ lny &= \ln \frac{(2x+1)^2}{(x+2)^3} \\ lny &= \ln(2x+1)^2 - \ln(x+2)^3 \\ lny &= 2\ln(2x+1) - 3\ln(x+2) \\ \frac{y'}{y} &= 2 \times \frac{1}{2x+1} \times 2 - 3 \times \frac{1}{x+2} \\ y' &= y \left(2 \times \frac{1}{2x+1} \times 2 - 3 \times \frac{1}{x+2} \right) \\ y' &= \frac{(2x+1)^2}{(x+2)^3} \left(\frac{4}{2x+1} - \frac{3}{x+2} \right) \\ y' &= \frac{(2x+1)^2}{(x+2)^3} \left(\frac{5-2x}{(2x+1)(x+2)} \right) \\ y' &= \frac{(2x+1)(5-2x)}{(x+2)^4} \end{aligned}$$

$$\begin{aligned} y &= \ln(\sin x) & y &= \ln(xsinx) \\ y' &= \frac{1}{\sin x} (\cos x) & y' &= \frac{1}{x \sin x} (1(\sin x) + (-\cos x)(x)) \\ y' &= \cot x & y' &= \frac{\sin x - x \cos x}{x \sin x} \end{aligned}$$