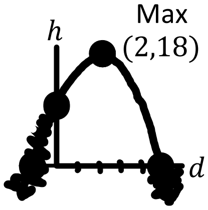
C12 - 3.4 - Projectile/#'s/Revenue Max/Min Notes

The height vs distance of a rock thrown off a cliff. What is the max height and the distance it took to get there?



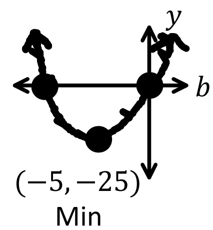
$$h = -2d^2 + 8d + 10$$
$$h' = -4d + 8$$
$$0 = -4d + 8$$
$$\ldots$$
$$d = 2\,m \qquad h = 18\,m$$

The difference between two numbers is 10 if their product is a minimum.

Let $a = 1st\ \#$
Let $b = 2nd\ \#$
$\boxed{let\ y = minimum}$

$$a - b = 10$$
$$a - b = 10$$
$$+b \qquad +b$$
$$a = (10 + b)$$

$$a \times b = minimum$$
$$a \times b = \cancel{minimum}\ y$$
$$y = a \times b$$
$$y = a \times b$$
$$y = (10 + b) \times b$$
$$y = 10b + b^2$$
$$y = b^2 + 10b$$
$$y' = 2b + 10$$
$$0 = 2b + 10$$
$$b = -5$$



$$a = 10 + (-5)$$
$$a = 5$$

$1st\ \# = 5$
$2nd\ \# = -5$

The minimum product is -25.

$$a - b = 10$$
$$5 - (-5) = 10$$

$$min = a \times b$$
$$= 5 \times -5$$
$$= -25$$

Find Two numbers who sum to 8 and the sum of their squares is a minimum.

Let $a = 1st\ \#$
Let $b = 2nd\ \#$
$\boxed{let\ y = minimum}$

$$a + b = 8$$
$$-b \qquad -b$$
$$a = 8 - b$$
$$a = (8 - b)$$

$$a^2 + b^2 = minimum$$
$$a^2 + b^2 = \cancel{minimum}\ y$$
$$y = a^2 + b^2$$
$$y = a^2 + b^2$$
$$y = (8 - b)^2 + b^2$$
$$y = 64 - 16b + b^2 + b^2$$
$$y = 2b^2 - 16b + 64$$
$$y' = 4b - 16$$
$$0 = 4b - 16$$
$$b = 4$$

$$a = 8 - (4)$$
$$a = 4$$

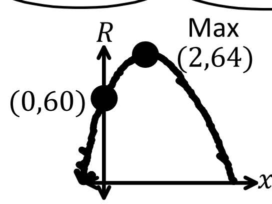
$a = 4$
$b = 4$

A student sells candy to their friends each day. Candy sells for 6 dollars, and 10 friends buy the candy. If they increase the price by 1 dollar, 1 less friend decides not to buy the candy. What is the price and quantity that will max revenue?

$$Let\ p = price, Let\ q = quantity, Let\ R = revenue$$
$$\boxed{Let\ x = \#\ of\ price\ increases}$$

$$P = 6 + 1t \qquad q = 10 - 1x$$

$$R = p \times q$$
$$R = (6 + 1x)(10 - 1x)$$
$$R = 60 - 6x + 10x - x^2$$
$$R = -x^2 + 4x + 60$$
$$R' = -2x + 4$$
$$0 = -2x + 4$$
$$x = 2$$

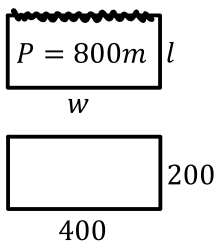


$$R = pq$$
$$R = 8 \times 8$$
$$R = \$64$$

$$p = 6 + 1(2) \qquad q = 10 - 1(2)$$
$$p = 6 + 2 \qquad q = 10 - 2$$
$$p = \$8 \qquad q = 8\ units$$

C12 - 3.4 - Rectangle/Prism/Cylinder Max/Min Notes

Max Area?



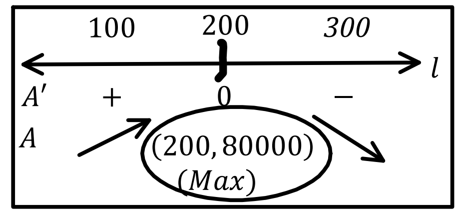
$$P = 2l + w$$
$$800 = 2l + w$$
$$w = 800 - 2l$$

$$w = 800 - 2(200)$$
$$w = 400\ m$$

$$800 = 2(200) + 400\ ✓$$

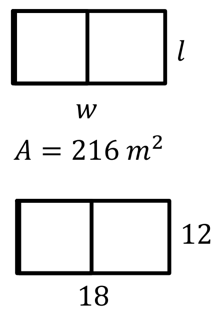
$$A = lw$$
$$A = l(800 - 2l)$$
$$A = -2l^2 + 800l$$
$$A' = -4l + 800$$
$$0 = -4l + 800$$
$$l = 200\ m$$

$$A = 400(200)$$
$$A = 80000\ m^2$$



Max Area is 80000 m^2.

Min Perimeter?



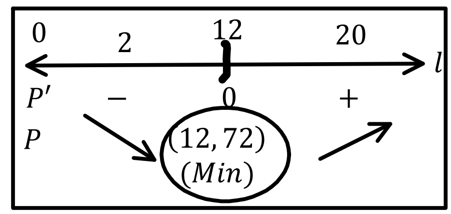
$$A = lw$$
$$216 = lw$$
$$w = \frac{216}{l}$$

$$w = \frac{216}{12}$$
$$w = 18\ m$$

$$216 = 12 \times 18\ ✓$$

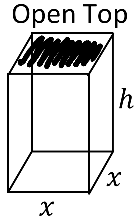
$$P = 3l + 2w$$
$$P = 3l + 2\left(\frac{216}{l}\right)$$
$$P = 3l + 432l^{-1}$$
$$P' = 3 - 432l^{-2}$$
$$0 = 3 - 432l^{-2}$$
$$3 = \frac{432}{l^2}$$
$$l^2 = 144$$
$$l = 12\ m$$

$$P = 3(12) + 2(18)$$
$$P = 72\ m$$



Min Perimeter is 72 m.

Open Top



$V = 500mL$

Min SA?

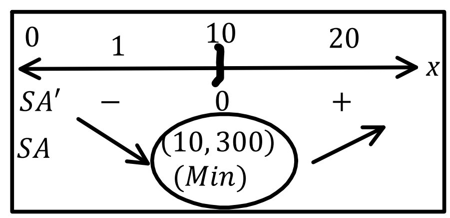
$\boxed{1mL = 1cm^3}$

$$V = lwh$$
$$500 = x^2h$$
$$h = \frac{500}{x^2}$$

$$h = \frac{500}{10^2}$$
$$h = 5\ cm$$

$$SA = x^2 + 4xh$$
$$SA = x^2 + 4x\left(\frac{500}{x^2}\right)$$
$$SA = x^2 + 2000x^{-1}$$
$$SA' = 2x - 2000x^{-2}$$
$$0 = 2x - 2000x^{-2}$$
$$2x = \frac{2000}{x^2}$$
$$x^3 = 1000$$
$$x = 10\ cm$$

$$SA = 10^2 + 4(10)(5)$$
$$SA = 300\ cm^2$$



Min SA is 300 cm^2.

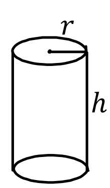
Cylinder V=1000mL, Find dimensions for Min SA.

$$V = \pi r^2 h$$
$$1000 = \pi r^2 h$$
$$h = \frac{1000}{\pi r^2}$$

$$h = \frac{1000}{\pi(5.4)^2}$$
$$h = 10.9\ cm$$

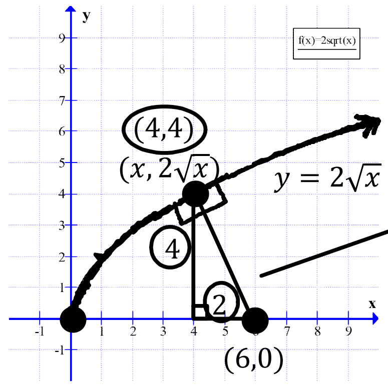
$$SA = 2\pi r^2 + 2\pi rh$$
$$SA = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$$
$$SA = 2\pi r^2 + 2000r^{-1}$$
$$SA' = 4\pi r - 2000r^{-2}$$
$$0 = 4\pi r - 2000r^{-2}$$
$$4\pi r = \frac{2000}{r^2}$$
$$\ldots$$
$$r = \sqrt[3]{\frac{500}{\pi}} = 5.4\ cm$$

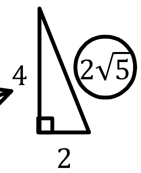
$$V = \pi r^2 h$$
$$V = \pi(5.4)^2(10.9)$$
$$V = 1000$$



C12 - 3.4 - Distance/to Curve/Ladder Max/Min Notes

Shortest Distance from $2\sqrt{x}$ to pt (6,0).





$a^2 + b^2 = c^2$

$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad c = \sqrt{a^2 + b^2}}$

$d = \sqrt{((4) - (6))^2 + ((4) - (0))^2}$ $\quad c = \sqrt{2^2 + 4^2}$

$d = \sqrt{4 + 16}$

$d = \sqrt{20}$ $\quad a^2 + b^2 = c^2$

$d = 2\sqrt{5}$ $\quad 2^2 + 4^2 = (2\sqrt{5})^2$

$20 = 20$

$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$

$d = \sqrt{((x) - (6))^2 + ((2\sqrt{x}) - (0))^2}$

$d = \sqrt{x^2 - 12x + 36 + 4x}$ $\quad \boxed{y = \sqrt{f(x)} \quad y' = \frac{f'(x)}{2\sqrt{f(x)}}}$

$d = \sqrt{x^2 - 8x + 36}$

$d = (x^2 - 8x + 36)^{\frac{1}{2}}$

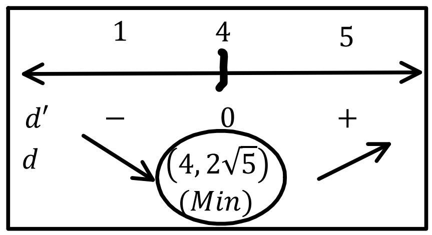
$d' = \frac{1}{2}(x^2 - 8x + 36)^{-\frac{1}{2}} \times (2x - 8)$

$d' = \frac{x - 4}{\sqrt{x^2 - 8x + 36}}$

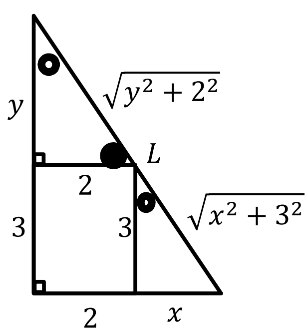
$0 = \frac{x - 4}{\sqrt{x^2 - 8x + 36}}$ $\quad y = 2\sqrt{x}$

$0 = x - 4$ $\quad y = 2\sqrt{4}$

$x = 4$ $\quad (4,4)$ $\quad y = 4$



Shortest ladder.



$L = \sqrt{(x + 2)^2 + (y + 3)^2}$

$L = \sqrt{(x + 2)^2 + \left(\left(\frac{6}{x}\right) + 3\right)^2}$

$L = \sqrt{x^2 + 4x + 4 + \frac{36}{x^2} + \frac{18}{x} + 9}$

...

$L' =$

$0 = 2x + 4 - 72x^{-3} - 18x^{-2}$

...

$0 = 2x^5 + 4x^3 - 18x - 72$

...

$x = 2.0596$

$\frac{y}{2} = \frac{3}{x}$

$y = \frac{6}{x}$

Not Optimal

$\boxed{L = \sqrt{y^2 + 2^2} + \sqrt{x^2 + 3^2}}$

...

$36\sqrt{x^2 - 9} = x^4\sqrt{36x^{-3} + 4}$

$L = \sqrt{(x + 2)^2 + \left(\left(\frac{6}{x}\right) + 3\right)^2}$

$L = \sqrt{(2.0596 + 2)^2 + \left(\left(\frac{6}{2.0596}\right) + 3\right)^2}$

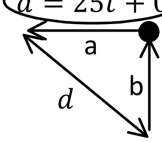
$L = 7.17$

Min Distance Between.

$\frac{da}{dt} = 25\frac{km}{hr}$

$d = 25t + 0$ $\quad \boxed{d_f = d_i + vt}$

Noon (a=0,b=40)



$\frac{db}{dt} = -20\frac{km}{hr}$

$d = -20t + 20$

$\boxed{y = mx + b}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(25t)^2 + (-20t + 20)^2}$

$d' = \frac{2(25t) + 2(-20t + 20)(-20)}{2\sqrt{(25t)^2 + (-20t + 20)^2}}$

...

$0 = 2(25t) + 2(-20t + 20)(-20)$

$0 = 50t + 800t - 800$

$t = 0.94\ hr$

$a = vt$ $\quad b = vt$

$a = 25(0.94)$ $\quad b = 40 - 20(0.94)$

$a = 23.5$ $\quad b = 22.2$

$c = \sqrt{a^2 + b^2}$

$c = 33.32\ km$

End Points

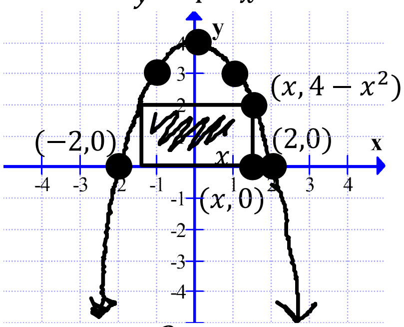
Noon (d=40km)

2pm (d=50km)

C12 - 3.4 - Area Under Graph/Shapes Max/Min Notes

Max rectangle area under $y = 4 - x^2$ above $x - axis$.

$y = 4 - x^2$



$A = lw$
$A = (2x)(y)$
$A = 2x(4 - x^2)$
$A = 8x - 2x^3$
$A' = 8 - 6x^2$
$0 = 8 - 6x^2$
$6x^2 = 8$
$x^2 = \frac{4}{3}$
$x = \pm\frac{2}{\sqrt{3}}$

$A = 2x(y)$
$A = 2\left(\frac{2}{\sqrt{3}}\right)\left(\frac{8}{3}\right)$
$A = \frac{32}{3\sqrt{3}}$

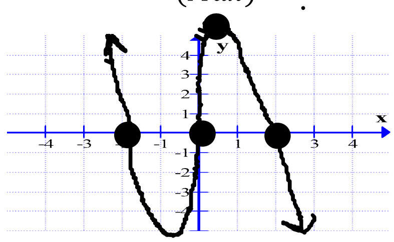
Domain
$x - int, y = 0$
$y = 4 - x^2$
$0 = 4 - x^2$
$x^2 = 4$
$x = \pm 2$
$(\pm 2, 0)$
$x \geq 0 \quad x \leq 2$
$0 \geq x \geq 2$
$y = 4 - x^2$
$y = 4 - \left(\frac{2}{\sqrt{3}}\right)^2$
$y = 4 - \frac{4}{3}$
$y = \frac{8}{3}$
$(\frac{2}{\sqrt{3}}, \frac{8}{3})$

$0.5 \quad \frac{2}{\sqrt{6}} = 0.81 \quad 1$

$A' \quad + \quad 0 \quad -$
$A \quad \nearrow \quad \left(\frac{2}{\sqrt{3}}, \frac{32}{3\sqrt{3}}\right) \quad \searrow$
(Max)

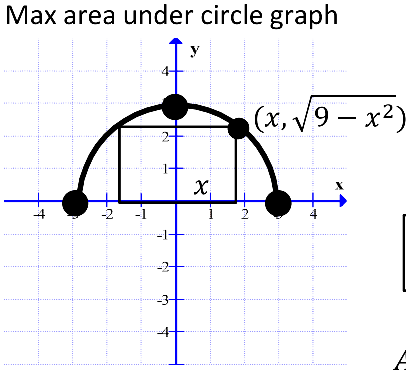
$A' = 8 - 6x^2$
$A' = 8 - 6\left(\frac{1}{2}\right)^2$
$A' = +ve$

$A' = 8 - 6x^2$
$A' = 8 - 6(1)^1$
$A' = -ve$

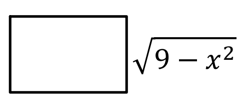


$A = 2x(2 + x)(2 - x)$

Max area under circle graph



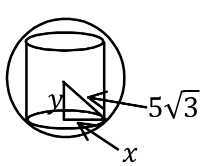
Equation of Circle
$x^2 + y^2 = r^2$
$x^2 + y^2 = 3^2$
$x^2 + y^2 = 9$
$y = \pm\sqrt{9 - x^2}$



$A = 2x\sqrt{9 - x^2}$
$A = 2(2.16)\sqrt{9 - (2.16)^2}$
$A = 9\ units^2$

$A = lw$
$A = 2x\sqrt{9 - x^2}$
$A' = 2\sqrt{9 - x^2} - \frac{2x}{\sqrt{9 - x^2}}(2x)$
$0 = 2\sqrt{9 - x^2} - \frac{4x^2}{\sqrt{9 - x^2}}$
$2\sqrt{9 - x^2} = \frac{4x^2}{\sqrt{9 - x^2}}$
$9 - x^2 = 2x$
...
$x = 2.16\ units \quad -4.16$

Max volume cylinder inscribed in sphere $r = 5\sqrt{3}$



$V = \pi r^2 h$
$V = \pi x^2(2y)$

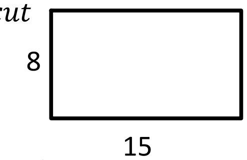
$V = \pi(75 - y^2)2y$
$V = \pi(150y - 2y^3)$
$V' = \pi(150 - 6y^2)$
$0 = \pi(150 - 6y^2)$
...
$y = 5$

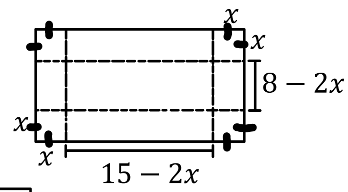
$x^2 + y^2 = r^2$
$x^2 + y^2 = (5\sqrt{3})^2$
$x^2 + y^2 = 75$
$x^2 = 75 - y^2$

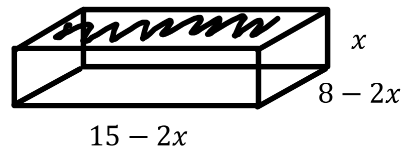
C12 - 3.4 - Open Box/String Max/Min Notes

An open rectangular box is made by cutting equal lengths from each corner of a 8 cm by 15 cm rectangular piece of cardboard, then folding up the sides. Find the length of the square that must be cut from each corner so the box has a Max Volume and find Max Volume.

$let\ x = length\ to\ cut$



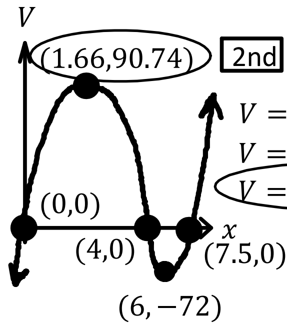




$V = lwh$
$V = (8-2x)(15-2x)x$
$V = 4x^3 - 46x^2 + 120x$
$V' = 12x^2 - 92x + 120$
$0 = 12x^2 - 92x + 120$
$0 = 3x^2 - 23x + 40$
$0 = (3x-5)(x-6)$
$x = \frac{5}{3} = 1.66 \quad x = 6$

$Domain:$
$0 > x > 4$

x cant be negative!
Cant cut two 4's off a 8!



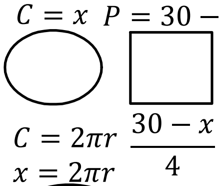
$\boxed{2nd}\ \boxed{Calc}\ \boxed{Max}$

$V = (8-2x)(15-2x)x$
$V = (8-2(1.67))(15-2(1.67)(1.67)$
$V = 90.74\ cm^3$

A 30 cm string cut to make a square and a circle. Find where to cut to max area.

$\underbrace{\quad}_{x}\overbrace{\underbrace{\qquad\quad}_{30-x}}^{30} \quad 0 \le x \le 30$

$C = x \quad P = 30 - x$



$C = 2\pi r$
$x = 2\pi r$
$r = \frac{x}{2\pi}$

$A = \pi r^2$
$A = \pi\left(\frac{x}{2\pi}\right)^2$
$A = \frac{x^2}{4\pi}$

$A = lw$
$A = \left(\frac{30-x}{4}\right)\left(\frac{30-x}{4}\right)$
$A = \frac{900-60x+x^2}{16}$

$A_T = \frac{x^2}{4\pi} + \frac{1}{16}(900-60x+x^2)$
$A_T' = \frac{1}{2\pi}x + \frac{1}{16}(-60+2x)$
$0 = \frac{x}{2\pi} - \frac{15}{4} + \frac{1}{8}x$
...
$x = 2.21\ cm$

$A_T = \frac{x^2}{4\pi} + \frac{1}{16}(900-60x+x^2)$
$A_T = \frac{2.21^2}{4\pi} + \frac{1}{16}(900-60(2.21)+(2.21)^2)$
$A_T = 16.2\ cm^2$

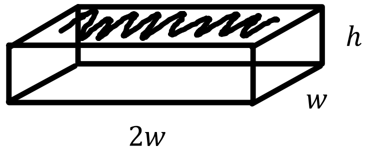
All Circle
$C = 2\pi r$
$30 = 2\pi r$
$r = \frac{30}{2\pi}$
$r = \frac{15}{\pi}$

$A = \pi r^2$
$A = \pi\left(\frac{15}{\pi}\right)^2$
$A = \frac{225}{\pi} = 71.7\ cm^2$

Don't cut it. All circle.
Check your endpoints.

All Square
$A = lw$
$A = \left(\frac{30}{4}\right)\left(\frac{30}{4}\right)$
$A = 56.25\ cm^2$

C12 - 3.4 - Cost Area/Length Max/Min Notes



$V = 8m^3$
$L = 2w$

$Cost_{Base} = \frac{\$4.5}{m^2}$ $\quad Cost_{Sides} = \frac{\$6}{m^2}$

$V = Lwh$
$8 = 2w^2h$
$4 = w^2h$
$h = \frac{4}{w^2}$

$h = \frac{4}{2^2}$
$h = 1m$

$SA^* = 2w^2 + 6wh$

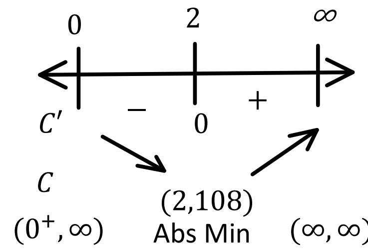
$SA = 2w^2 + 6w\left(\frac{4}{w^2}\right)$
$SA = 2w^2 + 24w^{-1}$
$C = 2w^2 \times 4.5 + 24w^{-1} \times 6$
$C = 9w^2 + 144w^{-1}$
$C' = 18w - 144w^{-2}$
$0 = 18w(1 - 8w^{-3})$

$\cancel{w = 0}$ $\quad 1 - 8w^{-3} = 0$

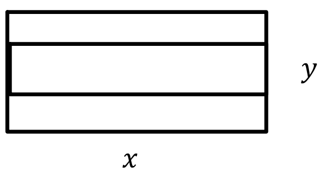
$1 = \frac{8}{w^3}$
$w = 2m$

$\boxed{Cost = Area \times \frac{Cost}{Area}}$

$C = 9w^2 + 144w^{-1}$
$C = 9(2)^2 + 144(2)^{-1}$
$C = \$108$



$\boxed{\text{Domain } 0 > x > \infty}$

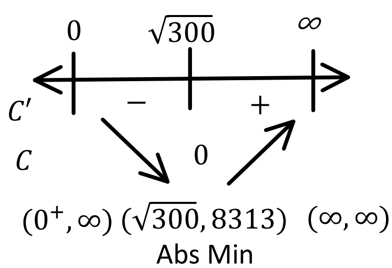


$A^* = 600m^2$ $\quad C = \frac{\$60}{m}$

$A = lw$
$A = xy$
$600 = xy$
$y = \frac{600}{x}$

$P = 2y + 4x$
$C = 2y \times 60 + 4x \times 60$
$C = 2\left(\frac{600}{x}\right) \times 60 + 4x \times 60$
$C = \frac{72000}{x} + 240x$
$C = 72000x^{-1} + 240x$
$C' = -72000x^{-2} + 240$
$C' = -\frac{72000}{x^2} + 240$
$0 = -\frac{72000}{x^2} + 240$
$\frac{72000}{x^2} = 240$
$x = \sqrt{300}$
$x = 10\sqrt{3}m$

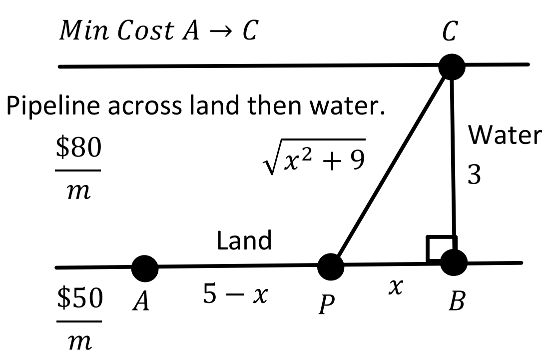
$\boxed{Cost = Length \times \frac{Cost}{length}}$



$y = \frac{600}{\sqrt{300}}$
$y = \frac{600}{10\sqrt{3}}$
$y = \frac{60}{\sqrt{3}}$
$y = 20\sqrt{3}\,m$

Check Answer

$A = xy$
$A = 20\sqrt{3} \times 10\sqrt{3}$
$A = 600$



$C = 50(5 - x) + 80\sqrt{x^2 + 9}$
$C' = -50 + \frac{80(2x)}{2\sqrt{x^2+9}}$
$0 = \cdots$
$50 = \frac{80x}{\sqrt{x^2+9}}$
$8x = 5\sqrt{x^2 + 9}$
$64x^2 = 25x^2 + 225$
$\cdots$
$x = 0.416$

$\boxed{Cost = length \times \frac{cost}{length}}$

Check End Points

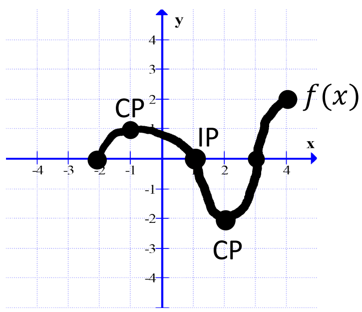
$C = 50(5 - x) + 80\sqrt{x^2 + 9}$
$C = 50(5 - 0) + 80\sqrt{0^2 + 9}$
$C = \$290$

$C = 50(5 - x) + 80\sqrt{x^2 + 9}$
$C = 50(5 - 5) + 80\sqrt{5^2 + 9}$
$C = \$466.48$

$C = 50(5 - x) + 80\sqrt{x^2 + 9}$
$C = 50(5 - 0.416) + 80\sqrt{0.416^2 + 9}$
$C = \$229.20$

C12 - 3.0 - f,f',f'' Graph Notes

$let\ m = slope$



A function
ie. Position

Find $f(x)$
intervals of :
-increase
-decrease.

$f(x)\ m > 0$

$f(x)\ Inc : f'(x) > 0$

$Inc : (-2,1)U(2,4)$

$f(x)\ m < 0$

$f(x)\ Dec : f'(x) < 0$

$Dec : (-1,2)$

Find where
$f(x)$ has a
relative :
-maximum
-minimum

$f(x)\ Max : f'(x) = 0$
$\&\ f'(x) > 0 \rightarrow f'(x) < 0$

$f(x)\ Max : (-1,1)$

$f(x)\ Min : f'(x) = 0$
$\&\ f'(x) < 0 \rightarrow f'(x) > 0$

$f(x)\ Min\ : (2,-2)$

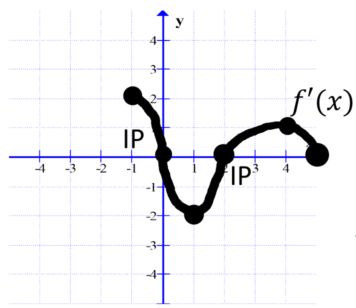
Find where
$f(x)$ is
concave :
-up
-down

$f(x)\ Conc\ up : f"(x) > 0$

$Conc\ up : (1,3)$

$f(x)\ Conc\ down : f"(x) > 0$

$Conc\ Down : (-2,1)U(3,4)$



A derivative function
ie. Velocity

Find $f(x)$
intervals of :
-increase
-decrease.

$f(x)\ Inc : f'(x) > 0$

$Inc : (-1,0)U(2,5)$

$f(x)\ Dec : f'(x) < 0$

$Dec : (0,2)$

Find where
$f(x)$ has a :
-maximum
-minimum

$f'(x) = 0\ \&$

$f(x) : Max$

$f'(x) > 0 \rightarrow f'(x) < 0$

$x = 0$

$f(x) : Min$

$f'(x) > 0 \rightarrow f'(x) < 0$

$x = 2$

Find where
$f(x)$ is
concave :
-up
-down

$f(x)\ Conc\ up :$
$f"(x) > 0$

$f'(x)\ m > 0$

$Conc\ up : (1,4)$

$f(x)\ Conc\ down :$
$f"(x) < 0$

$f'(x)\ m < 0$

$Conc\ down : (-1,1)U(4,5)$

Find $f'(x)$
intervals of :
-increase
-decrease.

$f'(x)\ m > 0$

$f'(x)\ Inc : f''(x) > 0$

$Inc : (1,4)$

$f'(x)\ m < 0$

$f'(x)\ Dec : f''(x) < 0$

$Dec : (-1,1)U(4,5)$

Find out where
$f(x)$ has a
point of
inflection.

$f"(x) = 0\ \&$

$f"(x) > 0 \rightarrow f''(x) < 0$ OR $f"(x) < 0 \rightarrow f''(x) > 0$

OR

$f'(x)\ m > 0 \rightarrow f'(x)\ m < 0$

$f'(x)\ max$

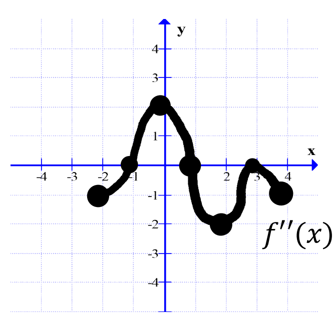
$x = 4$

OR

$f'(x)\ m < 0 \rightarrow f'(x)\ m > 0$

$f'(x)\ min$

$x = 1$



A Second derivative function
ie. Acceleration

Find out where
$f(x)$ has a
point of
inflection.

$f"(x) = 0\ \&$

$f"(x) > 0 \rightarrow f''(x) < 0$

$x = 1$

$f"(x) < 0 \rightarrow f''(x) > 0$

$x = -1$

Find where
$f(x)$ is concave
-up
-down

$f(x)\ Conc\ up :$
$f"(x) > 0$

$Conc\ up : (-1,1)$

$f(x)\ Conc\ down :$
$f"(x) < 0$

$Conc\ down : (-2,-1)U(1,3)U(3,4)$