

C12 - 5.10 - Trig Int Notes

$\int \cos 2x \, dx = \frac{\sin 2x}{2} + C$ Think: What would you have to divide by to reverse chain rule? **Reverse Chain** $\int \sin 2x \, dx = -\frac{\cos 2x}{2} + C$

$\int \cos 2x \, dx = \int \cos u \frac{du}{2}$ $u = 2x$ $\frac{du}{dx} = 2$ $dx = \frac{du}{2}$

$\int \cos u \frac{du}{2} = \frac{1}{2} \int \cos u \, du = \frac{1}{2} \sin u + C = \frac{1}{2} \sin 2x + C$

$\int \sin^2 x \, dx = \int \frac{1 - \cos 2x}{2} \, dx = \frac{1}{2} \int (1 - \cos 2x) \, dx = \frac{1}{2} \left(x - \frac{\sin 2x}{2} \right) + C$

Identities:
 $\cos 2x = 1 - 2 \sin^2 x$
 $\sin^2 x = \frac{1 - \cos 2x}{2}$
 $\cos 2x = 2 \cos^2 x - 1$
 $\cos^2 x = \frac{1 + \cos 2x}{2}$

$\int \cos^2 x \, dx = \int \frac{1 + \cos 2x}{2} \, dx = \frac{1}{2} \int (1 + \cos 2x) \, dx = \frac{1}{2} \left(x + \frac{\sin 2x}{2} \right) + C$

$\int \sin^2 x \cos x \, dx$ $u = \sin x$ $\frac{du}{dx} = \cos x$ $dx = \frac{du}{\cos x}$

$\int u^2 \cos x \frac{du}{\cos x} = \int u^2 \, du = \frac{u^3}{3} + C = \frac{\sin^3 x}{3} + C$

$\int \cos^3 x \, dx = \int \cos^2 x \cos x \, dx = \int (1 - \sin^2 x) \cos x \, dx = -\cot x - x + C$

$\int (\cos x - \sin^2 x \cos x) \, dx = -\sin x - \frac{\sin^3 x}{3} + C$

$\int \frac{\cos^2 x}{\sin^2 x} \, dx = \int (\csc^2 x - 1) \, dx = -\cot x - x + C$

Identities:
 $\cos^2 x = 1 - \sin^2 x$
 $\frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x} = \csc^2 x - 1$

$\int \frac{1}{1 + \sin x} \, dx$ **Conjugate** $\frac{1 + \sin x}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x} = \frac{1 - \sin^2 x}{1 - \sin x}$ **Separate Fractions** $\frac{1 - \sin^2 x}{1 - \sin x} = \frac{\cos^2 x}{1 - \sin x} = \frac{1}{\sec^2 x - \sec x \tan x}$

$\frac{\sin x}{\cos x} \times \frac{1}{\cos x} = \tan x \sec x$ $\frac{ab}{bc} = \frac{a}{b} \times \frac{b}{c}$

$\int \frac{1}{1 + \sin x} \, dx = \int \frac{1 - \sin x}{1 - \sin^2 x} \, dx = \int \frac{1 - \sin x}{\cos^2 x} \, dx = \int \frac{1}{\cos^2 x} \, dx - \int \frac{\sin x}{\cos^2 x} \, dx = \int \sec^2 x \, dx - \int \sec x \tan x \, dx = \tan x - \sec x + C$

$\int \cos^4 x \, dx = \int (\cos^2 x)^2 \, dx = \int \left(\frac{1 + \cos 2x}{2} \right)^2 \, dx = \frac{1}{4} \int (\cos 2x + 1)(\cos 2x + 1) \, dx = \frac{1}{4} \int (\cos^2 2x + 2\cos 2x + 1) \, dx = \frac{1}{4} \left(\frac{1}{2} \left(x + \frac{\sin 4x}{4} \right) + \sin 2x + x \right) + C = \frac{3x}{8} + \frac{\sin 4x}{32} + \frac{\sin 2x}{4} + C$

$\int \cos^2 2x \, dx = \int \frac{1 + \cos 4x}{2} \, dx = \frac{1}{2} \left(x + \frac{\sin 4x}{4} \right) + C$

$\int \frac{\sec x}{\tan^2 x} \, dx = \int \frac{\cos x}{\sin^2 x} \, dx = \int \frac{\cos x}{u^2} \frac{du}{\cos x} = \int \frac{1}{u^2} \, du = \int u^{-2} \, du = \frac{u^{-1}}{-1} + C = -\frac{1}{u} + C = -\frac{1}{\sin x} + C = -\csc x + C$

Trig Identities
 $\frac{1}{\frac{\cos x}{\sin^2 x}} = \frac{\sin^2 x}{\cos x} = \frac{1}{\cos x} \times \frac{\cos^2 x}{\sin^2 x}$
 $u = \sin x$
 $\frac{du}{dx} = \cos x$
 $dx = \frac{du}{\cos x}$