C12 - 5.11 - Trig Inv Comp ☐ Int Notes

Check : Take the Derivative

$$\int \frac{5}{1+x^2}\,dx$$

$$5\int \frac{1}{1+x^2}\,dx$$

$$5\tan^{-1}x + C$$

$$\int \frac{1}{9+9x^2}\,dx$$

$$\int \frac{1}{9(1+x^2)}\,dx$$

$$\frac{1}{9}\int \frac{1}{1+x^2}\,dx$$

$$\frac{1}{9}\tan^{-1}x + C$$

$$\int \frac{1}{1+4x^2}\,dx$$

$$\int \frac{1}{1+(2x)^2}\,dx$$

$$\frac{1}{2}\tan^{-1}2x + C$$

$$\int \frac{1}{1+u^2}\frac{du}{2}$$

$$\frac{1}{2}\tan^{-1}u + C$$

$$u = 2x$$

$$\frac{du}{dx} = 2$$

$$dx = \frac{du}{2}$$

$$\int \frac{1}{1+\left(\frac{x}{3}\right)^2}\,dx$$

$$3\tan^{-1}\left(\frac{x}{3}\right) + C$$

$$u = \frac{x}{3}$$

Reverse Chain

$$\int \frac{1}{9+x^2}\,dx$$

$$\frac{1}{9}\int \frac{1}{1+\left(\frac{x}{3}\right)^2}\,dx$$

$$\frac{1}{9}\times 3\tan^{-1}\left(\frac{x}{3}\right) + C$$

$$\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

$$9+x^2$$

$$9\left(1+\frac{x^2}{9}\right)$$

Factor

$$9\left(1+\left(\frac{x}{3}\right)^2\right)$$

Separate Exponents

$$\int \frac{1}{\sqrt{1-4x^2}}\,dx$$

$$\int \frac{1}{\sqrt{1-(2x)^2}}\,dx$$

$$\frac{1}{2}\sin^{-1}(2x) + C$$

Reverse Chain

$$\int \frac{1}{\sqrt{1-u^2}}\frac{du}{2}$$

$$\frac{1}{2}\sin^{-1}u + c$$

$$u = 2x$$

$$\frac{du}{dx} = 2$$

$$dx = \frac{du}{2}$$

$$\int \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dx$$

$$2\sin^{-1}\left(\frac{x}{2}\right) + C$$

$$u = \frac{x}{2}$$

Reverse Chain

$$\int \frac{1}{\sqrt{4-x^2}}\,dx$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\left(1-\left(\frac{x}{2}\right)^2\right.}}\,dx$$

$$\frac{1}{2}2\sin^{-1}\left(\frac{x}{2}\right) + C$$

$$\sin^{-1}\left(\frac{x}{2}\right) + C$$

$$\sqrt{4-x^2}$$

$$\sqrt{4\left(1-\frac{x^2}{4}\right)}$$

$$2\sqrt{1-\left(\frac{x}{2}\right)^2}$$

$$\int \frac{1}{\sqrt{4-x^2}}\,dx$$

$$\int \frac{1}{\sqrt{2^2-x^2}}\,dx$$

$$\sin^{-1}\left(\frac{x}{2}\right) + C$$

$$\int \frac{1}{\sqrt{2-x^2}}\,dx$$

$$\int \frac{1}{\sqrt{1-(x-1)^2}}\,dx$$

$$\sin^{-1}(x+1) + C$$

$$2x - x^2$$

$$-(x^2-2x)$$

Complete Square

$$-(x^2-2x+1-1)$$

$$-(x^2-2x+1)+1$$

$$-(x-1)^2+1$$

$$1-(x-1)^2$$

$$\int \frac{1}{9+36x^2}\,dx$$

$$\int \frac{1}{9(1+4x^2)}\,dx$$

$$\frac{1}{9}\int \frac{1}{1+(2x)^2}\,dx$$

$$\frac{1}{9}\frac{\tan^{-1}2x}{2} + C$$

$$\frac{1}{18}\tan^{-1}2x + C$$

$$\int \frac{1}{9+36x^2}\,dx$$

$$\int \frac{1}{3^2+(6x)^2}\,dx$$

$$\frac{1}{3}\frac{\tan^{-1}\left(\frac{6x}{3}\right)}{6} + C$$

$$\frac{1}{18}\tan^{-1}2x + C$$

$$\int -\frac{1}{\sqrt{1-16x^2}}\,dx$$

$$\int -\frac{1}{\sqrt{1-(4x)^2}}\,dx$$

$$\frac{\cos^{-1}4x}{4} + C$$

$$\int \frac{1}{5x\sqrt{25x^2-1}}\,dx$$

$$\int \frac{1}{5x\sqrt{(5x)^2-1}}\,dx$$

$$\frac{\sec^{-1}5x}{5} + C$$

$$\int -\frac{1}{1+36x^2}\,dx$$

$$\int -\frac{1}{1+(6x)^2}\,dx$$

$$\frac{\cot^{-1}6x}{6} + C$$

$$\int \frac{4}{1+4x^2}\,dx$$

$$4\int \frac{1}{1+(2x)^2}\,dx$$

$$4\times\frac{\tan^{-1}2x}{2} + C$$

$$2\tan^{-1}2x + C$$

$$\int \frac{4x^2}{1+4x^2}\,dx$$

$$\int \frac{4x^2+1-1}{1+4x^2}\,dx$$

$$\int \frac{4x^2+1}{1+4x^2}\,dx - \int \frac{1}{1+4x^2}\,dx$$

$$\int 1\,dx - \int \frac{1}{1+4x^2}\,dx$$

$$x - \frac{\tan^{-1}2x}{2}$$

$$\int \frac{4x}{1+4x^2}\,dx$$

$$4\int \frac{x}{u}\frac{du}{8x}$$

$$\frac{1}{2}\int \frac{1}{u}\,du$$

$$\frac{1}{2}lnu + C$$

$$\frac{1}{2}\ln(1+4x^2) + C$$

$$u = 1+4x^2$$

$$\frac{du}{dx} = 8x$$

$$dx = \frac{du}{8x}$$