C12 - 5.12 - Int by Parts Notes

$$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu \qquad \textit{Definite Integrals}$$

$$\int uv' = uv - \int vu'$$

Set it up $u = x$, $u' = 1dx$; $v =$, $v' = e^x dx$

1. u 4. v
3. u' 2. v'

Steps (7)

$$\int xe^x dx = xe^x - \int e^x dx$$
$$= xe^x - e^x + C$$

$u = x$, $u' = 1dx$; $v = e^x$, $v' = e^x dx$

u:LIPET:v

u: Logs-Inverse Trig-Poly-Exp-Trig:v

$y = xe^x - e^x + C$
$y' = e^x + xe^x - e^x$
$y' = xe^x$

Check : Take the derivative

Proof

$(uv)' = u'v + uv'$ *Product Rule*

$\int (uv)' = \int (u'v + uv')dx$ *Integrate Both Sides*

$\int (uv)' = \int u'v\,dx + \int uv'dx$ *Sum Rule*

$uv = \int u'v\,dx + \int uv'dx$ $\int (uv)' = uv$

$\int uv'dx = uv - \int u'v\,dx$ *Rearrange*

$\int lnx\,dx$ $u = lnx$ $v = x$

$xlnx - \int x\frac{1}{x}dx$ $du = \frac{1}{x}dx$ $dv = dx$

$xlnx - \int 1\,dx$

$xlnx - x + C$

$y = xlnx - x$
$y' = lnx + 1 - 1$
$y' = lnx$

$\int cos^4 x\,dx$ $u = \cos^3 x\,dx$ $v = sinx$

$\int cos^3 xcosx\,dx$ $\frac{du}{dx} = -3\cos^2 xsinx$ $v' = cosxdx$

$\cos^3 xsinx - \int -3cos^2 x\,sin^2 x\,dx$

$\cos^3 xsinx + \frac{3}{8}\left(x - \frac{sin4x}{4}\right) + C$

$\sin^2 xcos^2 x = (sinxcosx)^2$

$\left(\frac{sin2x}{2}\right)^2$

$\frac{1}{4}\sin^2 2x$

$\frac{1}{4}\left(\frac{1 - \cos(2(2x))}{2}\right)$

$\frac{1 - cos4x}{8}$

$sin2x = 2sinxcosx$
$cos2\theta = 1 - 2\sin^2\theta$
$\sin^2\theta = \frac{1 - cos2\theta}{2}$

$\int xe^{3x}dx$ $\int x^2e^x dx$ $\int e^x sinxdx$

Repeat Parts

let $m = ?$