

C12 - 5.13 - Int Partial Frac Notes

$a \neq b$

$$\int \frac{5x + 8}{(x + 3)(x + 2)} dx = ?$$

$$\frac{A}{x + 3} + \frac{B}{x + 2} = \frac{5x + 8}{(x + 3)(x + 2)}$$

$$\frac{qx + r}{(x - a)(x - b)} = \frac{A}{x - a} + \frac{B}{x - b}$$

$$\begin{aligned} A(x + 2) + B(x + 3) &= 5x + 8 \\ Ax + 2A + Bx + 3B &= 5x + 8 \\ Ax + Bx + 2A + 3B &= 5x + 8 \\ (A + B)x + (2A + 3B) &= 5x + 8 \end{aligned}$$

Multiply both sides by the LCD
Group/Factor

Add Fractions

$$\frac{\frac{7}{x + 3} - \frac{2}{x + 2}}{7(x + 2) - 2(x + 3)} = \frac{7(x + 2) - 2(x + 3)}{7x + 14 - 2x - 6} = \frac{5x + 8}{(x + 3)(x + 2)}$$

$$\int \left(\frac{7}{x + 3} - \frac{2}{x + 2} \right) dx = 7\ln(x + 3) - 2\ln(x + 2) + C$$

$$\begin{aligned} A + B &= 5 & 2A + 3B &= 8 \\ A &= 5 - B & 2(5 - B) + 3B &= 8 \\ A &= 5 - (-2) & 10 - 2B + 3B &= 8 \\ A &= 7 & B &= -2 \end{aligned}$$

$$y = 7\ln(x + 3) - 2\ln(x + 2) + C$$

$$y' = \frac{7}{x + 3} - \frac{2}{x + 2} \quad \text{Check: Take Derivative}$$

$$\int \frac{2x^2 - 7x - 6}{x(x + 1)(x - 2)} dx = ?$$

$$\frac{A}{x} + \frac{B}{x + 1} + \frac{C}{x - 2} = \frac{2x^2 - 7x - 6}{x(x + 1)(x - 2)}$$

$$\begin{aligned} \frac{A(x + 1)(x - 2) + Bx(x - 2) + Cx(x + 1)}{x(x + 1)(x - 2)} &= \frac{2x^2 - 7x - 6}{x(x + 1)(x - 2)} \\ Ax^2 - Ax - 2A + Bx^2 - 2Bx + Cx^2 + Cx &= 2x^2 - 7x - 6 \\ (A + B + C)x^2 + (-A - 2B + C)x - 2A &= 2x^2 - 7x - 6 \end{aligned}$$

$$\int \left(\frac{3}{x} + \frac{1}{x + 1} - \frac{2}{x - 2} \right) dx = 3\ln x + \ln(x + 1) - 2\ln(x - 2) + C$$

$$\begin{aligned} A + B + C &= 2 & -A - 2B + C &= -7 & -2A &= -6 \\ (3) + B + C &= 2 & -(3) - 2B + C &= -7 & A &= 3 \\ B + C &= -1 & -2B + C &= -4 & & \end{aligned}$$

$$y = 3\ln x + \ln(x + 1) - 2\ln(x - 2) + C$$

$$y' = \frac{3}{x} + \frac{1}{x + 1} - \frac{2}{x - 2} \quad \text{Check: Take Derivative}$$

$$\frac{\frac{3}{x} + \frac{1}{x + 1} - \frac{2}{x - 2}}{3(x + 1)(x - 2) + 1x(x - 2) - 2x(x + 1)} = \frac{3x^2 - 3x - 6 + x^2 - 2x - 2x^2 - 2x}{x(x + 1)(x - 2)} = \frac{2x^2 - 7x - 6}{x(x + 1)(x - 2)}$$

$$\begin{aligned} (1) + C &= -1 & B + C &= -1 \\ C &= -2 & -2B + C &= -4 \\ & & 3B &= 3 \\ & & B &= 1 \end{aligned}$$

$$\frac{px^2 + qx + r}{(x - a)(x - b)(x - c)} = \frac{A}{x - a} + \frac{B}{x - b} + \frac{C}{x - c}$$

$$\int \frac{x + 1}{x^2(x - 1)} dx = ?$$

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x - 1} = \frac{x + 1}{x^2(x - 1)}$$

Add Fractions

$$\begin{aligned} Ax(x - 1) + B(x - 1) + Cx^2 &= x + 1 \\ Ax^2 - Ax + Bx - B + Cx^2 &= x + 1 \\ (A + C)x^2 + (-A + B)x - B &= x + 1 \end{aligned}$$

$$\frac{-\frac{2}{x} - \frac{1}{x^2} + \frac{2}{x - 1}}{-2x(x - 1) - 1(x - 1) + 2x^2} = \frac{-2x^2 + 2x - x + 1 + 2x^2}{x^2(x - 1)} = \frac{x + 1}{x^2(x - 1)}$$

$$\int \left(-\frac{2}{x} - \frac{1}{x^2} + \frac{2}{x - 1} \right) dx = -2\ln x + \frac{1}{x} + 2\ln(x - 1) + C$$

$$\begin{aligned} A + C &= 0 & -A + B &= 1 & -B &= 1 \\ (-2) + C &= 0 & -A + (-1) &= 1 & B &= -1 \\ C &= 2 & -A - 1 &= 1 & A &= -2 \end{aligned}$$

Check: Take Derivative

$$y = -2\ln x + \frac{1}{x} + 2\ln(x - 1) + C$$

$$y' = -\frac{2}{x} - \frac{1}{x^2} + \frac{2}{x - 1}$$

$$\frac{px + q}{(x - a)^2} = \frac{A}{x - a} + \frac{B}{(x - a)^2}$$

$$\frac{px^2 + qx + r}{(x - a)(x^2 + bx + c)} = \frac{A}{x - a} + \frac{Bx + C}{x^2 + bx + c}$$