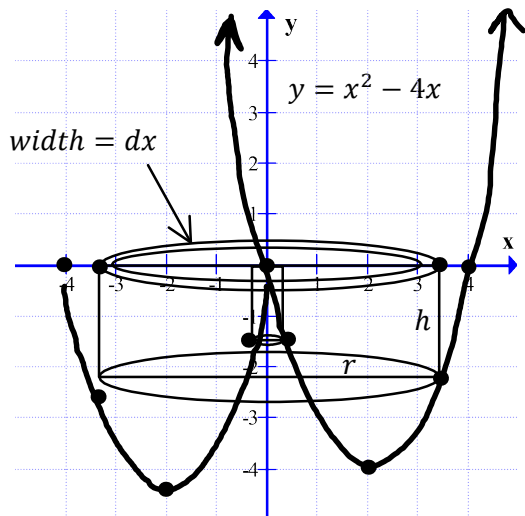


C12 - 5.14 - Cylindrical Shells Int Notes

Revolution about y-axis $V = 2\pi \int_a^b xy dx$ $V = 2\pi \int_a^b rh dx$ $r, x = \text{shell radius}$
 $h, y = \text{shell height}$

$SA = Ch$
 $SA = 2\pi rh$



$$V = 2\pi \int_a^b xy dx$$

X-LAND
(y-axis)

$$y = x^2 - 4x$$

$$y = x(x - 4)$$

$$V = 2\pi \int_0^4 x(x^2 - 4x) dx$$

$$x = 0 \quad x = 4$$

$$V = 2\pi \int_0^4 (x^3 - 4x^2) dx$$

$$V = 2\pi \left(\frac{x^4}{4} - \frac{4x^3}{3} \right) \Big|_0^4$$

$$V = 2\pi \left(\left(\frac{4^4}{4} - \frac{4(4)^3}{3} \right) - (0) \right)$$

$$V = 2\pi \left(64 - \frac{256}{3} \right) = -\frac{128\pi}{3} = -134.04$$

$$V = \pi \int_0^4 ((r_{outer})^2 - (r_{inner})^2) dx = \pi \int_0^4 \left((\sqrt{y+4} + 2)^2 - (-\sqrt{y+4} + 2)^2 \right) dy$$

Y-LAND
(y-axis)

$$a^2 - b^2 = (a+b)(a-b) = \pi \int_0^4 (4 + 2\sqrt{y+4}) dy$$

$$y = x^2 - 4x$$

$$y = (x^2 - 4x + 4) - 4$$

$$y = (x - 2)^2 - 4$$

$$= \pi \left(4y + \frac{4(y+4)^{3/2}}{3} \right) \Big|_0^4$$

$$= 111.53$$

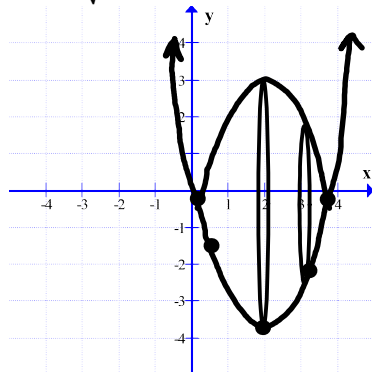
$$\pm\sqrt{y+4} = x - 2$$

$$x = \pm\sqrt{y+4} + 2$$

Complete Square

Revolution about x-axis $V = 2\pi \int_a^b yx dy$ $V = 2\pi \int_a^b rh dx$ $r, y = \text{shell radius}$
 $h, x = \text{shell height}$

$$x = -\sqrt{y+4} + 2 \quad x = +\sqrt{y+4} + 2$$



$$V = 2\pi \int_a^b yx dy$$

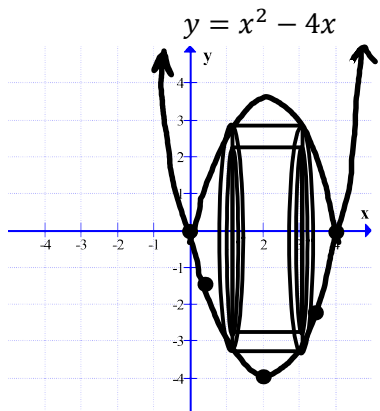
Y-LAND
(x-axis)

$$V = 2\pi \int_2^4 y(\sqrt{y+4} + 2) dy$$

$$V = 175.848$$

$$V = 175.848 \times 2$$

$$V = 351.699$$



$$V = \pi \int_0^4 (x^2 - 4x)^2 dx$$

$$V = \pi \int_0^4 (x^4 - 8x^3 + 16x^2) dx$$

$$V = \pi \left(\frac{x^5}{5} - 2x^4 + \frac{16x^3}{3} \right) \Big|_0^4$$

$$V = 107.23$$