

C12 - 5.0 - Exp Growth

$$\frac{dy}{dt} = ky$$

$$\frac{dy}{y} = kdt$$

$$\int \frac{1}{y} dy = \int kdt$$

Separate Variables
Integrate Both Sides
Initial Condition

$$\ln_e y = kt + C$$

$$y = e^{kt+C}$$

$$y = e^{kt} \times e^C$$

$$y = e^C e^{kt}$$

$$y = Ae^{kt}$$

Separate Exponents
 $A = e^C$

$A = \text{Present Value}$
 $y = \text{Future Value}$
 $k = \text{Constant Rate}$
 $t = \text{time}$

Change Forms

In both sides
Exponent In Front
Divide
Change of Base
Divide
OR! $\ln e = 1$
Change Forms

Cont!

$$y = y_0 e^{kt}$$

$$\frac{2}{4\text{hrs}} = 1e^{4k}$$

$$\ln 2 = \ln e^{4k}$$

$$\ln 2 = 4k \ln e$$

$$y_0 = 1 \quad \frac{\ln 2}{\ln e} = 4k$$

$$\ln_e 2 = 4k$$

$$k = \frac{\ln_e 2}{4}$$

$$k = 0.173$$

Prop!

$$y = 4, t = 0$$

$$y_0 = 4$$

$$y = 10, t = 2$$

$$? y, t = 5$$

$$10 = 4e^{2k}$$

$$\frac{5}{2} = e^{2k}$$

$$\dots$$

$$\ln_e \left(\frac{5}{2}\right) = 2k$$

$$k = \frac{\ln_e \left(\frac{5}{2}\right)}{2}$$

$$y = 4e^{\left(\frac{\ln_e \left(\frac{5}{2}\right)}{2}\right)(5)}$$

$$y = 39.53$$

Prop! $\frac{\times 2}{5\text{hrs}}$

$$y = y_0 e^{kt}$$

$$2 = 1e^{5k}$$

$$\dots$$

$$k = \frac{\ln_e 2}{5}$$

$$y = y_0 e^{kt}$$

$$5 = 1e^{\left(\frac{\ln_e(2)}{5}\right)t}$$

$$\dots$$

$$\ln 5 = \left(\frac{\ln_e(2)}{5}\right) t \ln e$$

$$t = \frac{5 \ln_e 5}{\ln_e 2}$$

$$t = 11.61\text{hrs}$$

Prop! $\frac{1}{2} \text{life} = 5730\text{yrs}$? $t, y = 1\%y_0$

$$y = y_0 e^{kt}$$

$$\frac{1}{2} = 1e^{5730k}$$

$$\dots$$

$$\ln_e \left(\frac{1}{2}\right) = 5730k$$

$$k = \frac{\ln_e \left(\frac{1}{2}\right)}{5730}$$

$$y = y_0 e^{kt}$$

$$0.01 = 1e^{\frac{\ln_e \left(\frac{1}{2}\right)}{5730} t}$$

$$\dots$$

$$\ln_2 0.01 = \frac{\ln_e \left(\frac{1}{2}\right)}{5730} t \ln e$$

$$t = 5730 \frac{\ln_e 0.01}{\ln_e \left(\frac{1}{2}\right)} \text{yrs}$$

$$t = 38069.30 \text{yrs}$$