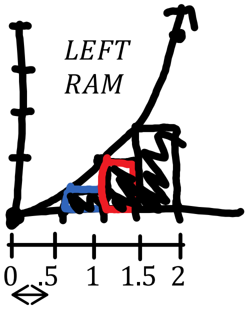
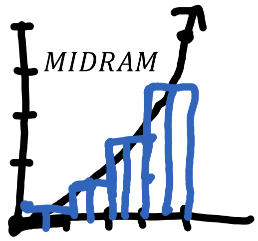
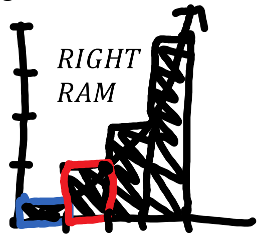
C12 - 5.2 - Riemann's Sums Integration Notes

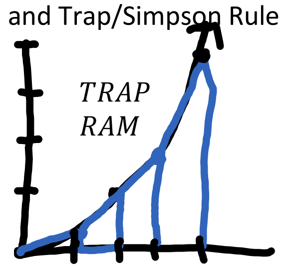
$\boxed{A = xy}$

Find area under* $y = x^2$ from 0-2 using four (n=4) rectangles. LRAM, MRAM & RRAM, and Trap/Simpson Rule .









$$h = horizontal\ width = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$$

$\boxed{\begin{array}{l} Height\ is: LEFT\ y, \\ MID\ y, RIGHT\ y, values^*. \end{array}}$

$$\boxed{A_{TRAP} = \left(\frac{y_n + y_{n+1}}{2}\right)h}$$

Average heights × width

$$A = lw + lw + lw + lw$$
$$A = w(l + l + l + l)$$
$$A = \frac{1}{2}\left(0 + \frac{1}{4} + 1 + \frac{9}{4}\right)$$
$$A = \frac{14}{8} = 1.75$$

Underestimate

$$A = lw + lw + lw + lw$$
$$A = w(l + l + l + l)$$
$$A = \frac{1}{2}\left(\frac{1}{16} + \frac{9}{16} + \frac{25}{16} + \frac{49}{16}\right)$$
$$A = \frac{21}{8} = 2.625$$

Increasing and Concave Up*

$$A = lw + lw + lw + lw$$
$$A = w(l + l + l + l)$$
$$A = \frac{1}{2}\left(\frac{1}{4} + 1 + \frac{9}{4} + 4\right)$$
$$A = \frac{15}{4} = 3.75$$

Overestimate

x	y
0	0
$0.5 = 1$	$0.25 = 1/4$
1	1
$1.5 = 3$	$2.25 = 9/4$
2	4

$$A_{TRAP} = \frac{h}{2}(y_0 + 2y_1 + 2y_2 + \cdots 2y_{n-1} + y_n)$$
$$A = \frac{\frac{1}{2}}{2}\left(0 + 2\left(\frac{1}{4}\right) + 2(1) + 2\left(\frac{9}{4}\right) + 4\right)$$
$$A = \frac{11}{4} = 2.75$$

$$A_{TRAP} = \frac{LRAM + RRAM}{2}$$
$$A_{TRAP} = \frac{\frac{14}{8} + \frac{15}{4}}{2} = \frac{11}{4} = 2.75$$

x	y
$0.25 = 1/4$	$0.0625 = 1/16$
$0.75 = 3/4$	$0.5625 = 9/16$
$1.25 = 5/4$	$1.5625 = 25/16$
$1.75 = 7/4$	$3.0625 = 49/16$

$$\boxed{A = \int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x}$$

$$\boxed{x_i = a + i\Delta x}$$

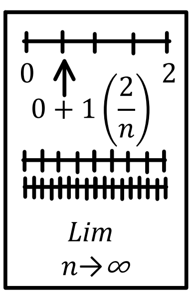
$$x_k = 0 + k\left(\frac{2}{n}\right)$$
$$x_k = \frac{2k}{n}$$

$$\boxed{\Delta x = \frac{b-a}{n}}$$
$$\Delta x = \frac{2-0}{n}$$
$$\Delta x = \frac{2}{n}$$

Simpsons Rule

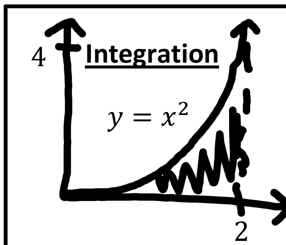
$$A_{SIMP} = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + \cdots 2y_{n-2} + 4y_{n-1} + y_n)$$
$$A = \frac{\frac{1}{2}}{3}\left(0 + 4\left(\frac{1}{4}\right) + 2(1) + 4\left(\frac{9}{4}\right) + 4\right)$$
$$A = \frac{8}{3} = 2.\overline{6}$$

$$A = \int_0^2 x^2 dx = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{2k}{n}\right)\left(\frac{2}{n}\right)$$
$$A \approx \frac{2}{n}\sum_{i=1}^{n}\left(\frac{2k}{n}\right)^2$$
$$A \approx \frac{2}{n}\sum_{i=1}^{n}\frac{4k^2}{n^2}$$
$$A \approx \frac{8}{n^3}\sum_{i=1}^{n}k^2$$
$$A \approx \frac{8}{n^3}\,\frac{n(n+1)}{2}\,\frac{(2n+1)}{3}$$
$$A \approx \frac{8}{n^3}\,\frac{n(n+1)}{2}\,\frac{(2n+1)}{3}$$
$$A \approx \frac{8n^2 + 12n + 4}{3n^2}$$
$$A = \lim_{n\to\infty} \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2}$$
$$A = \frac{8}{3} = 2.\overline{6}$$



$$A = \sum_{i=1}^{n} i^2 = \frac{n(n+1)}{2}\,\frac{(2n+1)}{3}$$
$$A = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
$$A = \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$
$$A = \sum_{i=1}^{n} c = nc$$

Integration



$$A = \int_0^2 x^2 dx = \left[\frac{x^{2+1}}{2+1}\right]_0^2$$
$$= \left[\frac{x^3}{3}\right]_0^2$$
$$= \frac{(2)^3}{3} - \frac{(0)^3}{3}$$
$$= \frac{8}{3} = 2.\overline{6}$$

Math 9