C12 - 5.4 - Integration Notes
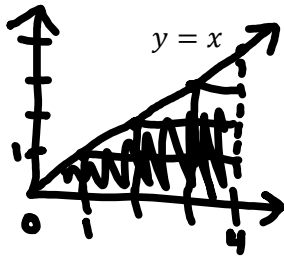
Find Area under* $y = x$; $0 \le x \le 4$
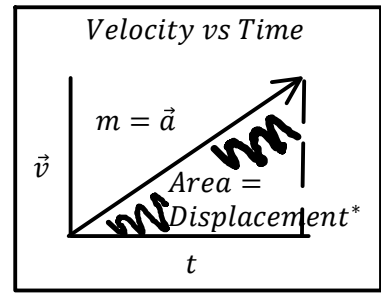

$$A = \int_a^b f(x)dx \quad \boxed{FTC}$$

$$A = \int_0^4 xdx = \frac{x^2}{2}\Big|_0^4$$
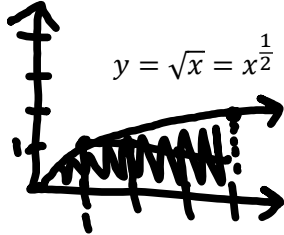
$$= \frac{(4)^2}{2} - \frac{(0)^2}{2} = 8$$

Check by Geometry

$$A = \frac{bh}{2}$$

$$A = \frac{4 \times 4}{2}$$

$$\boxed{A = 8}$$

Check with math 9

Velocity vs Time

$m = \vec{a}$

$\vec{v}$

$Area = Displacement^*$

t

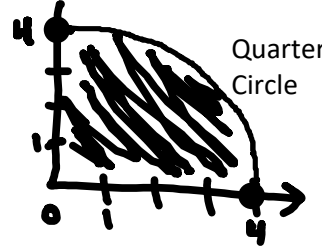Find Area under* $y = \sqrt{x}$; $0 \le x \le 4$

$y = \sqrt{x} = x^{\frac{1}{2}}$

$$\int_0^4 x^{\frac{1}{2}}dx = \frac{2x^{\frac{3}{2}}}{3}\Big|_0^4$$

$$= \frac{2(4)^{\frac{3}{2}}}{3} - \frac{(0)^{\frac{3}{2}}}{2}$$

$$= \frac{16}{3} = 5.33$$

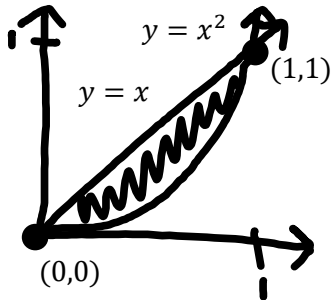Find Area under* $y = \sqrt{16 - x^2}$; $0 \le x \le 4$

Quarter Circle

$$A = \frac{\pi r^2}{4}$$

$$A = \frac{\pi(4)^2}{4}$$

$$\boxed{A = 4\pi}$$

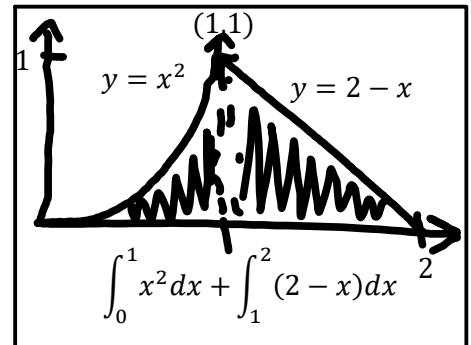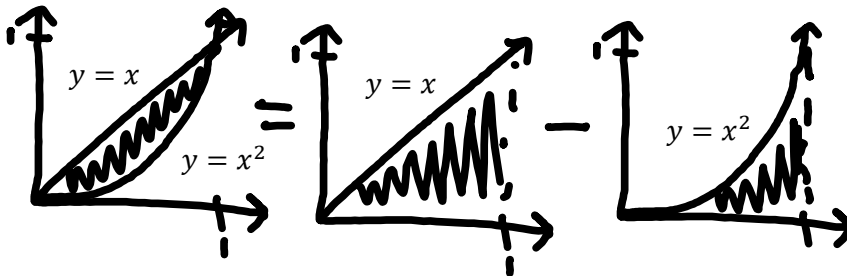
Find the area between the curves using Integration.

$y = x^2$

$y = x$

$(1,1)$

$(0,0)$

Find Intersections

$$x = x^2$$
$$x^2 - x = 0$$
$$x(x - 1) = 0$$

$\boxed{x = 0}$ $\boxed{x = 1}$

x	$y = x$	$y = x^2$
0	0	0
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$
1	1	1

$y = x$

$y = x^2$

$=$

$y = x$

$-$

$y = x^2$

$(1,1)$

$y = x^2$

$y = 2 - x$
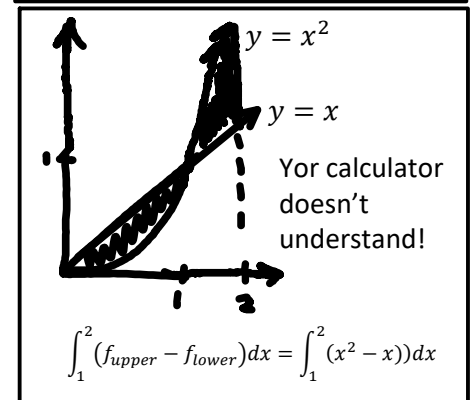
$$\int_0^1 x^2 dx + \int_1^2 (2 - x)dx$$

$$\int_0^1 (f_{upper} - f_{lower})dx = \int_0^1 (x - (x^2))dx$$

$$= \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1$$

$$= \frac{(1)^2}{2} - \frac{(1)^3}{3} - \left(\frac{(0)^2}{2} - \frac{(0)^3}{3}\right)$$

$$= \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$$

$y = x^2$

$y = x$

Yor calculator doesn't understand!

$$\int_1^2 (f_{upper} - f_{lower})dx = \int_1^2 (x^2 - x))dx$$

If either function is below the x-axis, subtracting a negative area adds the area. Don't forget to distribute any negatives.

1.333

−0.666

$$\int_0^2 (x^2 - 1)\,dx = 0.666$$

Enclosed Area=2

Signed Area=0.666 (M9)