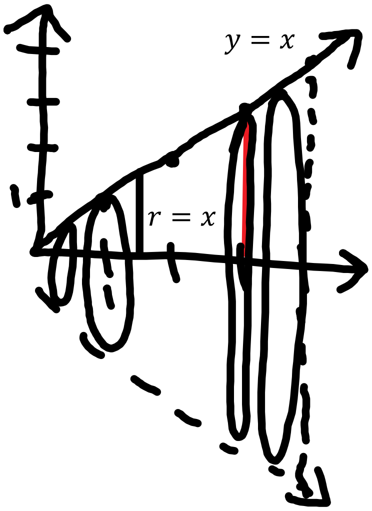
C12 - 5.5 - X-Land Vol Integration Notes

X-LAND

Find Volume of revolution of $y = x\,;\quad 0 \le x \le 4,$ about the $x = axis$.



$$V = \int_a^b A(x)dx = \int_a^b \pi r^2 dx$$

radius is the y height

$$= \pi \int_0^4 x^2 dx$$

$$= \pi \left. \frac{x^3}{3} \right|_0^4$$

$$= \pi \left(\frac{4^3}{3} - \frac{0^3}{3} \right) = \boxed{\frac{64\pi}{3}}$$

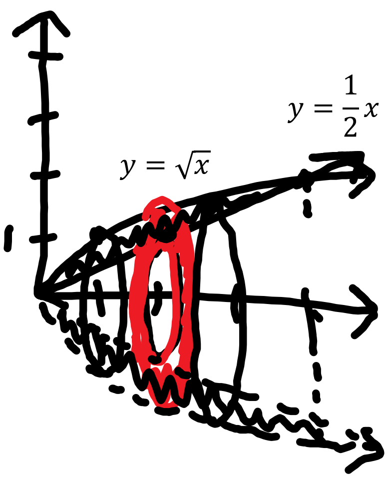
Check by Geometry

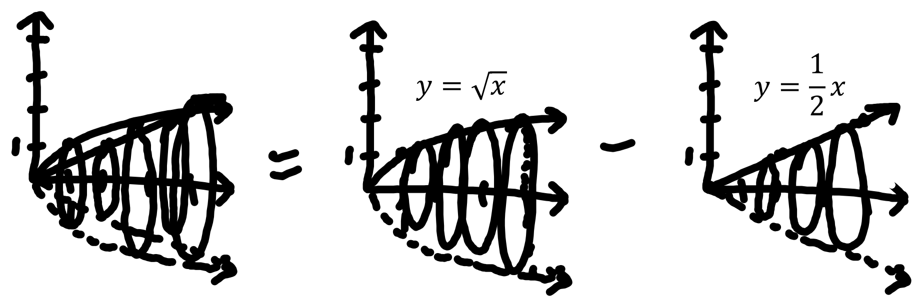
$$V_{cone} = \frac{1}{3}\pi r^2 h$$

$$V_{cone} = \frac{1}{3}\pi 4^2 4$$

$$\boxed{V_{cone} = \frac{64\pi}{3}}$$

Find Volume of revolution between $y = \sqrt{x}$ & $y = \frac{1}{2}x\,;\; 0 \le x \le 4$, around the $x - axis$.



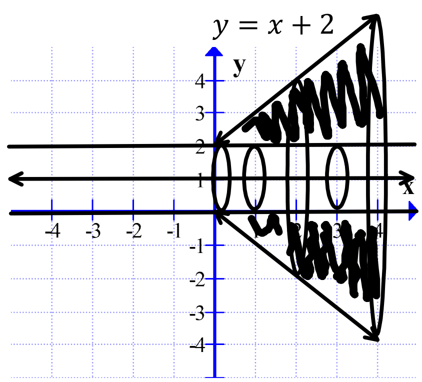


$$V = \pi \int_0^4 \left((r_{outer})^2 - (r_{inner})^2\right)dx = \pi \int_0^4 \left((\sqrt{x})^2 - \left(\frac{1}{2}x\right)^2 \right) dx$$

$$= \pi \int_0^4 \left(x - \frac{1}{4}x^2 \right) dx$$

$$= \pi \left. \left(\frac{x^2}{2} - \frac{x^3}{12} \right) \right|_0^4$$

$$= \pi \left(\frac{4^2}{2} - \frac{4^3}{12} - \left(\frac{0^2}{2} - \frac{0^3}{12} \right) \right) = \boxed{\frac{8\pi}{3}}$$



$0 \le x \le 4$

Outer

Inner

$y = 2$

$Axis: y = 1$

$$r^*_{outer} = f(x)_{outer} - Axis \qquad r^*_{inner} = f(x)_{inner} - Axis$$

$$r_{outer} = (x + 2 - (1)) \qquad r_{inner} = (2 - (1))$$

$$V = \pi \int_0^4 \left((r_{outer})^2 - (r_{inner})^2\right)dx = \pi \int_0^4 \left((x+1)^2 - (1)^2\right)dx$$

$$= \pi \int_0^4 (x^2 + 2x)dx$$

$$= \pi \left. \left(\frac{x^3}{3} + x^2 \right) \right|_0^4$$

$$= \pi \left(\frac{4^3}{3} + 4^2 - \left(\frac{0^3}{3} - 0^2 \right) \right) = \boxed{\frac{112\pi}{3}}$$

$$V = \frac{1}{3}\pi r^2 h - \frac{1}{3}\pi r^2 h - \pi r^2 h$$

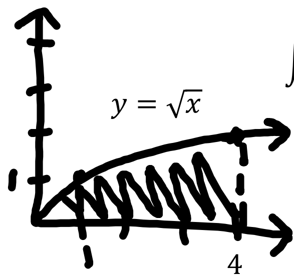
$$V = \frac{1}{3}\pi(5)^2(5) - \frac{1}{3}\pi(1)^2(1) - \pi(1)^2(4)$$

$$\boxed{V = \frac{112\pi}{3}}$$

C12 - 5.5 - Y-Land/Vol Int Notes

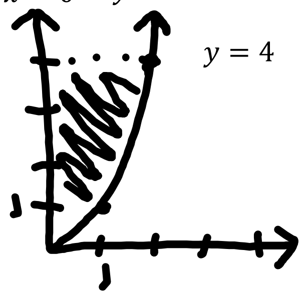
Y-LAND

Find the Area under* $y=\sqrt{x}$ $\quad 0 \le x \le 4$



$$\int_0^4 \sqrt{x}dx = \int_0^4 x^{\frac{1}{2}}dx$$
$$= \frac{2x^{\frac{3}{2}}}{3}\Big|_0^4$$
$$= \frac{2(4)^{\frac{3}{2}}}{3} - \frac{2(0)^{\frac{3}{2}}}{3}$$
$$= \frac{16}{3}$$

$x=0$ $\quad y=x^2$



Integrate with respect to y.

$$\int_0^4 \sqrt{y}dy = \int_0^4 y^{\frac{1}{2}}dy$$
$$= \frac{2y^{\frac{3}{2}}}{3}\Big|_0^4$$
$$= \frac{2(4)^{\frac{3}{2}}}{3} - \frac{2(0)^{\frac{3}{2}}}{3}$$
$$= \frac{16}{3}$$

$y=x^2$
$\sqrt{y}=\sqrt{x^2}$
$x=\pm\sqrt{y}$
$x=\sqrt{y}$

$4=x^2$
$\sqrt{4}=\sqrt{x^2}$
$x=\pm 2$
$(2,4)$

Find Intersection

OR

$$\int_0^2 (f_{upper}-f_{lower})dx = \int_0^2 (4-x^2)dx$$
$$= \left(4x - \frac{x^3}{3}\right)\Big|_0^2$$
$$= \left(4(2) - \frac{(2)^3}{3} - \left(4(0) - \frac{(0)^3}{3}\right)\right)$$
$$= 8 - \frac{8}{3}$$
$$= \frac{16}{3}$$

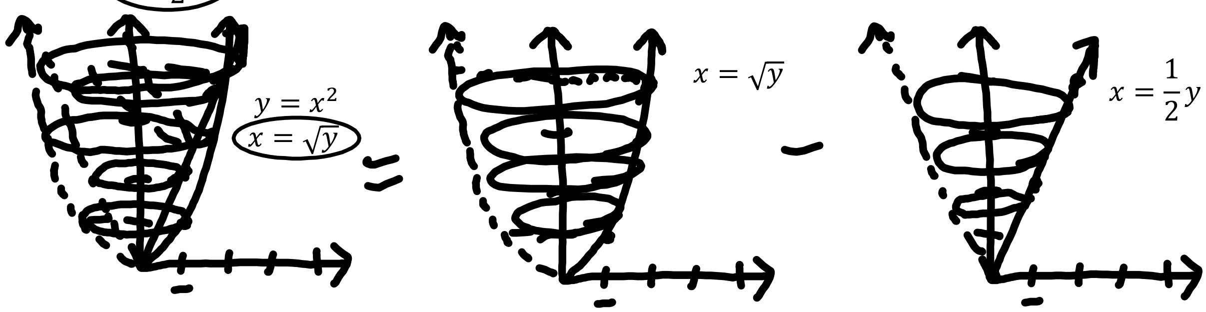
X-LAND

Isolate for x
Integrate
Find Interval (Intersections)

Rotate page 90 degrees Counter Clockwise and go from right to left.

Find the Volume of revolution between the two functions by Integration around the y axis.

$y=2x$
$x=\frac{1}{2}y$



$$V = \pi\int_0^4 \left((r_{outer})^2 - (r_{inner})^2\right)dx = \pi\int_0^4 \left((\sqrt{y})^2 - \left(\frac{1}{2}y\right)^2\right)dy$$
$$= \pi\int_0^4 \left(y - \frac{1}{4}y^2\right)dy$$
$$= \pi\left(\frac{y^2}{2} - \frac{y^3}{12}\right)\Big|_0^4$$
$$= \pi\left(\frac{4^2}{2} - \frac{4^3}{12} - \left(\frac{0^2}{2} - \frac{0^3}{12}\right)\right)$$
$$= \frac{8\pi}{3}$$

Find Intersections
$x^2 = 2x$
$x^2 - 2x = 0$
$x(x-2) = 0$
$x=0$ $\quad x=2$
$(0,0)$ $\quad (2,4)$