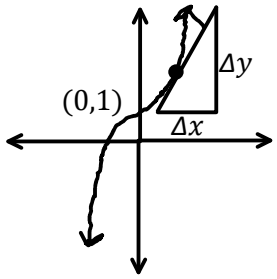
C12 - 5.6 - Initial Condition/Diff/Eq Int Notes

$y = x^3 + 1$



$rise = \Delta y = dy$

$run = \Delta x = dx$

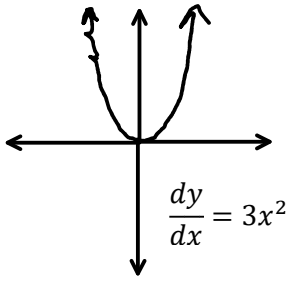
$$m = slope = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = f'(x) = y'$$

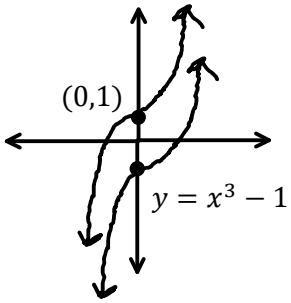
Δ – Infintesimally small change in

$$\lim_{\Delta x \to 0}$$

How do you know? $+C$

$y' = 3x^2$





$$\frac{dy}{dx} = 3x^2$$
$$dy = 3x^2\,dx$$
$$\int dy = \int 3x^2\,dx$$
$$y = x^3 + C$$

$y = x^3 + C$

$(1) = (0)^3 + C$ $(0,1)$

$C = 1$

$y = x^3 + 1$

Separate Variable
Integrate
Initial Condition
Solve for C
Write Equation
Possibly leave Implicitly

Find $f(x)$ $\quad f'(x) = 3x^2 \quad f(0) = 1$

$$f(x) = f(0) + \int_0^x f'(x)\,dx$$

$$F(b) = F(a) + \int_a^b f(x)\,dx$$

$$f(x) = 1 + \int_0^x 3x^2\,dt$$
$$f(x) = 1 + (x^3)\Big|_0^x$$
$$f(x) = 1 + (x^3 - 0^3)$$
$$f(x) = x^3 + 1$$

$$\frac{dy}{dx} = \frac{3x^3}{2y}$$
$$2ydy = 3x^2dx$$
$$\int 2ydy = \int 3x^2dx$$
$$y^2 = x^3 + C$$
$$y = +\!= \sqrt{x^3 + C}$$

$f(4) = 10$

$y = \pm\sqrt{x^3 + C}$

$10 = \pm\sqrt{(4)^3 + C}$

$100 = 64 + C$

$C = 36$

$y = \sqrt{x^3 + 36}$

A substance has a half-life of 50 years. How long to decay 400 g to 25 g?

$$\frac{dy}{dt} = ky$$
$$\frac{dy}{y} = kdt$$
$$\int \frac{1}{y}dy = \int kdt$$
$$lny = kt + C$$
$$y = e^{kt+C}$$
$$y = e^{kt} \times e^C$$
$$y = e^C e^{kt}$$
$$y = Ae^{kt}$$

Separate Variables
Integrate Both Sides
Initial Condition

Separate Exponents

$A = e^C$

$y = Ae^{kt}$

$400 = Ae^{4k}$

$400 = Ae^0$ $\quad f(0) = 400$

$A = 400$

$y = 400e^{kt}$

$A = Present\ Value$
$y = Future\ Value$
$k = Constant\ Rate$
$t = time$

$Half = 200$

$y = 400e^{kt}$

$200 = 400e^{50k}$

$0.5 = e^{50k}$

$ln0.5 = 50klne$

$ln0.5 = 50k$

$$k = \frac{ln0.5}{50}$$
$$k = \frac{ln2^{-1}}{50}$$
$$k = -\frac{ln2}{50}$$

$y = 400e^{-\frac{ln2}{50}t}$

$25 = 400e^{-\frac{ln2}{50}t}$

$0.125 = e^{-\frac{ln2}{50}t}$

$$ln0.125 = -\frac{ln2}{50}t$$

$t = 150\ yrs$