C12 - 5.9 - U Sub Integration Notes

$\int 2x(x^2+1)^2\,dx \quad u = x^2+1$

$\int (u)^2(2x)\frac{du}{2x} \quad \frac{du}{dx} = 2x$

$\int u^2\,du \quad dx = \frac{du}{2x}$

$\frac{u^3}{3} + C$

$\frac{(x^2+1)^3}{3} + C$

U Sub
- $-Choose\ u$
- $-\frac{du}{dx}, Isolate\ dx$
- $-Sub\ dx, Cancel$
- $-Integrate$
- $-Sub\ x\ back\ in\ for\ u$

Integration by "u" Substitution: Choose a "u" who's derivative is present, and cancels (Algebra*)

OR Expand

$y' = 2x(x^2+1)^2$

$y' = 2x(x^4+2x^2+1)$

$y' = 2x^5+4x^3+2x$

$y = \frac{2x^6}{6} + \frac{4x^4}{4} + \frac{2x^2}{2} + C$

$y = \frac{x^6}{3} + x^4 + x^2 + C$

$\int_1^2 2x(x^2+1)^2\,dx$ X LAND OR

$\int_1^2 (u)^2(2x)\frac{du}{2x}$ XU LAND U LAND

$\int_1^2 u^2\,du \longrightarrow \int_3^5 u^2\,du$

$u = x^2+1 \quad u = (2)^2+1 \quad u = 5$

$u = x^2+1 \quad u = (1)^2+1 \quad u = 3$

$\frac{u^3}{3}\Big|_1^2$ $\frac{u^3}{3}\Big|_3^5$

$\frac{(x^2+1)^3}{3}\Big|_1^2$ X LAND $\frac{(5)^3}{5} - \frac{(2)^3}{3}$

$\frac{((2)^2+1)^3}{3} - \frac{((1)^2+1)^3}{3}$ $\frac{117}{3}$

$\frac{117}{3}$

- $-Change\ Integral\ to\ u!$
- $-Sub\ Top - Bottom$
- $-Sub\ x\ back\ in\ for\ u$
- $-Sub\ Top - Bottom$

$\int \frac{2x}{1+x^2}dx \quad u = 1+x^2$

$\int \frac{2x}{u}\frac{du}{2x} \quad \frac{du}{dx} = 2x$

$\int \frac{1}{u}du \quad dx = \frac{du}{2x}$

$lnu + C$

$\ln(1+x^2) + C$

$\int \frac{2x+1}{1+x^2}dx$ Separate Fractions

$\int \frac{2x}{1+x^2}dx + \int \frac{1}{1+x^2}dx$

$\int \frac{2x}{u}\frac{du}{2x} + \int \frac{1}{1+x^2}dx$

$\int \frac{1}{u}du + \int \frac{1}{1+x^2}dx$

$lnu + \tan^{-1}x + C$

$\ln(1+x^2) + \tan^{-1}x + C$

$\int x\sqrt{x-1}\,dx \quad u = x-1$

$\int x\sqrt{u}\,du \quad \frac{du}{dx} = 1$

$\int (u+1)\sqrt{u}\,dx \quad dx = du$

$\int u^{\frac{3}{2}} + u^{\frac{1}{2}}\,du \quad u = x-1$

$x = u+1$

Algebra

$\frac{2u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3} + C$

$\frac{2(x-1)^{\frac{5}{2}}}{5} + \frac{2(x-1)^{\frac{3}{2}}}{3} + C$

$\int \frac{x}{x^2-4}dx \quad u = x^2-4$

$\int \frac{x}{u}\frac{du}{2x} \quad \frac{du}{dx} = 2x$

$\frac{1}{2}\int \frac{1}{u}\,du \quad dx = \frac{du}{2x}$

$\frac{1}{2}lnu + C$

$\frac{1}{2}\ln(x^2-4) + C$

$\int \frac{1}{xlnx}dx \quad u = lnx$

$\int \frac{1}{xu}\,xdu \quad \frac{du}{dx} = \frac{1}{x}$

$\int \frac{1}{u}du \quad dx = xdu$

$lnu + C$

$\ln(lnx) + C$

$\int \frac{lnx}{x}dx \quad u = lnx$

$\int \frac{u}{x}\,xdu \quad \frac{du}{dx} = \frac{1}{x}$

$\int udu \quad dx = xdu$

$\frac{u^2}{2} + C$

$\frac{\ln^2 x}{2} + C$

$\int xe^{x^2}dx \quad u = x^2$

$\int xe^u\frac{du}{2x} \quad \frac{du}{dx} = 2x$

$\frac{1}{2}\int e^u du \quad dx = \frac{du}{2x}$

$\frac{1}{2}e^u + C$

$\frac{1}{2}e^{x^2} + C$