M10 - 8.0 - Systems Intersections Notes

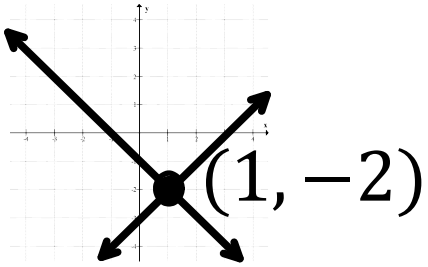
3 possible cases:
- one solution
- no solutions
- infinite number of solutions.

One Solution : Different slopes

$x - y - 3 = 0$
$+y \quad +y$
$x - 3 = y$
$y = x - 3$
$m = 1$

$y = -x - 1$
$m = -1$

Different Slopes



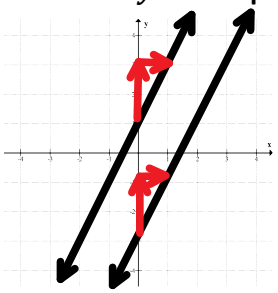
One Solution

No Solutions : Parallel Lines

$y = 2x - 3$
$m = 2$
$b = -3$

$y = 2x + 1$
$m = 2$
$b = 1$

Same slope
Different y-intercepts



No Solution

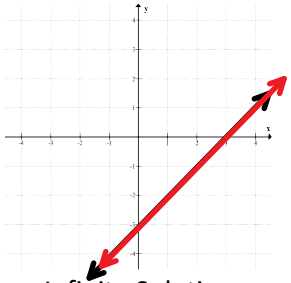
These Lines Never Intersect

Infinite Solutions : Same Line

$y = x - 3$
$m = 1$
$b = -3$

$y = x - 3$
$m = 1$
$b = -3$

Same slope
Same y-intercept



Infinite Solutions

These Lines are on Top of Each Other

Is (1,2) a point on the line?

$y = x + 1$
$(2) = (1) + 1$
$2 = 2$ ✓

Point on the Line

$(1,2)$
(x, y)

x	y
1	***2***

If it works it's a Point on the Line.

Is (1,2) a point on the line?

$y = -x + 3$
$(2) = -(1) + 3$
$2 = 2$ ✓

Point on the Line

$(1,2)$
(x, y)

x	
1	***2***

If it's is on both lines it must be the Intersection!

$x + y = 3$ Algebra $x + y = 3$
$-x \quad -x \quad (1) + (2) = 3$
$y = -x + 3 \quad 3 = 3$ ✓

Is (1,3) a point on the line?

$y = x + 1$
$(3) \neq (1) + 1$
$3 \neq 2$ ✗

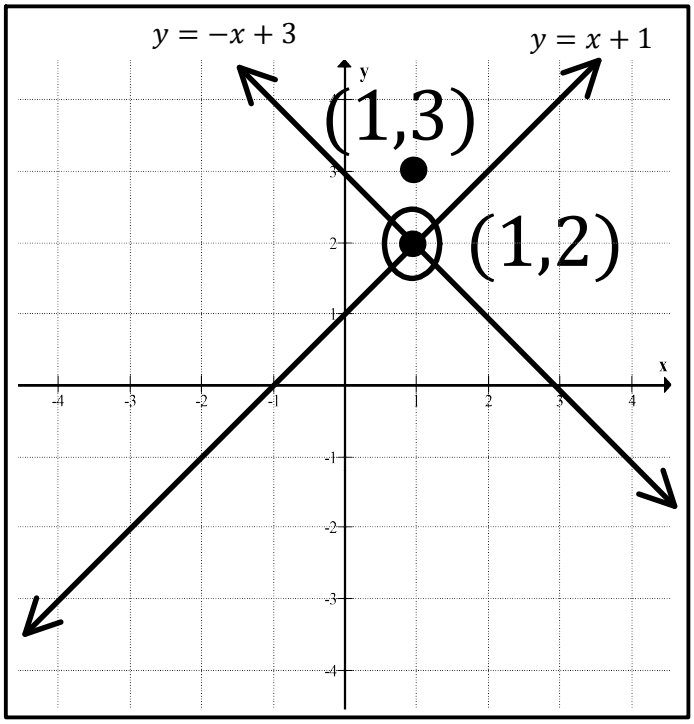
Point Not on Line

$(1,2)$
(x, y)

x	y
1	***2***
2	***3***
3	***4***

Therefore Not the intersection!

If it doesn't work it's NOT a Point on the Line.



One/None/Infinite Solutions
Point on/not on Line

M10 - 8.0 - Systems Notes

Solve by Substitution

$y = x + 1$ $\quad$ $y = -2x + 4$

$x + 1 = -2x + 4$

$-1 \quad\quad -1$

$x = -2x + 3$

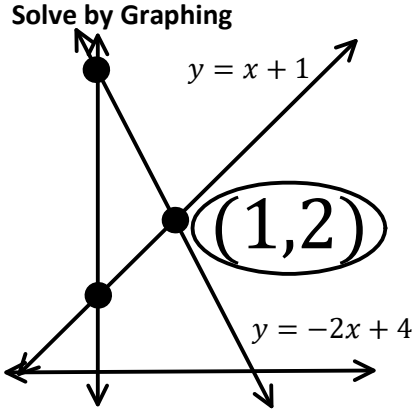
$y = x + 1 \quad +2x \quad +2x$

$y = (1) + 1 \quad \frac{3x}{3} = \frac{3}{3}$

$y = 2 \quad\quad x = 1$

$(1,2)$

Solve by Graphing



Solve by Elimination

$y = x + 1$

$-(y = -2x + 4)$

$0 = 3x - 3$

$+3 \quad\quad +3$

$\frac{3}{3} = \frac{3x}{3}$

$x = 1$

Substitution - Don't Need to Isolate

$x = (3 - y)$ $\quad$ $2y - 2(x) = 10$

$2y - 2(3 - y) = 10$

$2y - 6 + 2y = 10$

$4y - 6 = 10$

$+6 \quad +6$

$4y = 16$

$y = 4$

$x = 3 - y$

$x = 3 - (4)$

$x = -1$

$(-1,4)$

Substitution - Need to Isolate

$x + y = 11$ $\quad$ $2x - 2(y) = 6$

$-x \quad\quad -x$

$y = (11 - x)$ $\quad$ $2x - 2(11 - x) = 6$

$2x - 22 + 2x = 6$

$4x - 22 = 6$

$+22 \quad +22$

$4x = 28$

$\frac{4x}{4} = \frac{28}{4}$

$x = 7$

$y = 11 - x$

$y = 11 - 7$

$y = 4$

$(7,4)$

Solving by Elimination

$2y = x - 2$ $\quad$ $y = x - 3$

$-(y = x - 3)$

$y = 0 + 1$ $\quad$ $-2 - (-3) = 1$

$y = 1$

$y = x - 3$

$(1) = x - 3$

$+3 \quad\quad +3$

$4 = x$

$(4,1)$

Add or subtract to eliminate x *or* y.

$2x - 3y = 2$ $\quad$ $x + 2y = 8$

$-(2x + 4y = 16)$ $\quad$ $2(x + 2y = 8)$

$0x - 7y = -14$ $\quad$ $2x + 4y = 16$

$\frac{-7y}{-7} = -\frac{14}{-7}$

$y = 2$

Multiply/Divide

$x + 2y = 8$

$x + 2(2) = 8$

$x + 4 = 8$

$x = 4$

$(4,2)$

$3x + 2y = 6$ $\quad$ $-2x + 3y = 6$

$2(3x + 2y = 6)$ LCM $3(-2x + 3y = 6)$

$6x + 4y = 12$ $\quad$ $-6x + 9y = 18$

$6x + 4y = 12$

$+(-6x + 9y = 18)$

...

$y = -6x + 2$ $\quad$ $y + 4x = 0$

$+6x \quad +6x$

$y + 6x = 2$

$(y + 6x = 2)$

$-(y + 4x = 0)$

$0y + 2x = 2$

Or In Here

$2x = 2$

$\frac{2x}{2} = \frac{2}{2}$

$x = 1$

$(1, -4)$

$y + 4x = 0$

$y + 4(1) = 0$

$y + 4 = 0$

$-4 \quad -4$

$y = -4$

Line up Terms

$y + x = \#$

$y + x = \#$

For Example*

Substitution
Graphing
Isolations
Elimination
Add/Subtract Equations
Line up terms
Multiply/Divide

$3y + x = 4$

$-(3y + 2x = 18)$

$-x = -14$

$x = 14$

$0.5y + \frac{x}{3} = 3$

$\left(\frac{1}{2}y + \frac{x}{3} = 3\right) \times 6$ LCD

$3y + 2x = 18$

$3y + 2(14) = 18$

$3y + 28 = 18$

$3y = 18 - 28$

$3y = -10$

$\frac{3y}{3} = -\frac{10}{3}$

$y = -\frac{10}{3}$

$(14, -\frac{10}{3})$