

C12 - 11.0 - Combinatorics Review

Logic

Guess and Check

| | | |
|--------------------------------|---|---|
| Fundamental Counting Principle | $a \times b \times c$ | Factorial Notation! |
| Blanks | ___ , ___ , ___ | Repeats? $\times \div 2 \text{ or } \#!$ |
| | $\frac{\# \text{ options}}{\text{options}}$ | Replacement |
| | | Given! |
| Tree Diagrams | Multiply Branches | $(\text{outcomes per trial})^{\text{number of trials}}$ |
| Table | Add Leaves | |

All - None *Cannot = All - Can*

Cases! Cases: Multiply within cases, add separate cases.

Identical Objects $\frac{(\# \text{ of letters})!}{(\text{repeating letter})! (\text{other repeating letter})! \dots}$ Circle: $(n - 1)!$

| | | |
|---------------------------------|--------------------------------|--|
| Combinatorics Formulas | Permutations: Order Matters | Combinations: Doesn't Matter |
| $n \geq r$ | ${}_n P_r = \frac{n!}{(n-r)!}$ | ${}_n C_r = \frac{n!}{r!(n-r)!}$ ${}_n C_r = \frac{{}_n P_r}{r!}$ |
| n : # of objects to choose from | | $\binom{n}{r}$ |
| r : # of objects choosing | | |

Paths in Squares: $\frac{(l+w)!}{l!w!}$ *Paths in Cubes:* $\frac{(l+w+h)!}{l!w!h!}$

Binomial Theorem $t_{k+1} = {}_n C_k a^{n-k} b^k$; $(a+b)^n$; $n + 1$ terms

; k is always one less than the term number.

Probability $\frac{\# \text{ of}}{\text{Options}}$

C12 - 11.0 - Table of Cards

| Hearts ♥ | Diamonds ♦ | Spades ♠ | Clubs ♣ |
|----------|------------|----------|---------|
| Ace ♥ | Ace ♦ | Ace ♠ | Ace ♣ |
| 2 ♥ | 2 ♦ | 2 ♠ | 2 ♣ |
| 3 ♥ | 3 ♦ | 3 ♠ | 3 ♣ |
| 4 ♥ | 4 ♦ | 4 ♠ | 4 ♣ |
| 5 ♥ | 5 ♦ | 5 ♠ | 5 ♣ |
| 6 ♥ | 6 ♦ | 6 ♠ | 6 ♣ |
| 7 ♥ | 7 ♦ | 7 ♠ | 7 ♣ |
| 8 ♥ | 8 ♦ | 8 ♠ | 8 ♣ |
| 9 ♥ | 9 ♦ | 9 ♠ | 9 ♣ |
| 10 ♥ | 10 ♦ | 10 ♠ | 10 ♣ |
| Jack ♥ | Jack ♦ | Jack ♠ | Jack ♣ |
| Queen ♥ | Queen ♦ | Queen ♠ | Queen ♣ |
| King ♥ | King ♦ | King ♠ | King ♣ |



Ace is both high and low

52 card deck
4 suits
13 cards in each suit
4 of each rank

5 card poker hands ${}_{52}C_5 = 2598960 \text{ Hands}$ $P(\text{hand}) = \frac{\# \text{ of}}{{}_{52}C_5}$

| Hand | | | | | | | ${}_nC_r$ | # of |
|----------------|--------|--------|---------|--------|---------|-------------------------------|---|---------|
| Royal Flush | Ace ♥ | King ♥ | Queen ♥ | Jack ♥ | 10 ♥ | 10-Ace same suit | ${}_4C_1 \times 1 = 4$ | 4 |
| Straight Flush | 5 ♠ | 6 ♠ | 7 ♠ | 8 ♠ | 10 ♠ | 5 card run same suit | ${}_4C_1 \times 10 - 4 = 36$ | 36 |
| 4 of a Kind | 7 ♥ | 7 ♦ | 7 ♠ | 7 ♣ | 3 ♦ | 4 same rank, 1 other | ${}_{13}C_1 {}_4C_4 {}_{48}C_1$ | 624 |
| Full House | 2 ♥ | 2 ♦ | 2 ♠ | 4 ♦ | 4 ♣ | 3 same rank, 1 pair | ${}_{13}C_1 {}_4C_3 {}_{12}C_1 {}_4C_2$ | 3744 |
| Flush | 4 ♠ | 8 ♠ | Jack ♠ | 2 ♠ | 6 ♠ | All same suit, no straight | ${}_4C_1 \times {}_{13}C_5 - 40$ | 5108 |
| Straight | 3 ♥ | 4 ♣ | 5 ♦ | 6 ♠ | 7 ♣ | 5 card run, not all same suit | $({}_4C_1)^5 \times 10 - 40$ | 10200 |
| 3 of a kind | 9 ♥ | 9 ♦ | 9 ♠ | 2 ♦ | 5 ♠ | 3 kind, 2 others not a pair | ${}_{13}C_1 {}_4C_3 {}_{12}C_2 ({}_4C_1)^2$ | 54912 |
| 2 pair | 4 ♥ | 4 ♠ | 5 ♦ | 5 ♠ | Queen ♠ | 2 different pairs, 1 other | ${}_{13}C_2 ({}_4C_2)^2 {}_{44}C_1$ | 123552 |
| Pair | King ♦ | King ♠ | 6 ♥ | 9 ♠ | 2 ♦ | 1 pair 3 others | ${}_{13}C_1 {}_4C_2 {}_{12}C_3 ({}_4C_1)^3$ | 1098240 |
| High Card | Jack ♠ | 8 ♦ | 4 ♠ | 2 ♦ | 7 ♠ | None of the above | ${}_{52}C_5 - \text{above sum}$ | 1302540 |

3 Kind ≠

$${}_{13}C_1 {}_4C_3 {}_{12}C_1 {}_4C_1 {}_{11}C_1 {}_4C_1$$

Pair ≠

$${}_{13}C_1 {}_4C_2 {}_{12}C_1 {}_4C_1 {}_{11}C_1 {}_4C_1 {}_{10}C_1 {}_4C_1$$

Note:

$${}_{48}C_1 = {}_{12}C_1 {}_4C_1$$

2 Pair ≠

$${}_{13}C_1 {}_4C_2 {}_{12}C_1 {}_4C_2 {}_{11}C_1 {}_4C_1$$

$${}_{44}C_1 = {}_{11}C_1 {}_4C_1$$