C12 - 3.0 - Poly Notes

$$\boxed{\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\text{dividend}} \end{array}}$$

$\boxed{\text{The factor is the only opposite*}}$

Long/Synthetic Division :

$$\frac{x^2+5x+6}{x+3} = x+2$$

$$\begin{array}{rl} & x+2 \\ x+3 & \overline{)\,x^2+5x+6} \\ - & \underline{x^2+3x} \\ & 2x+6 \\ - & \underline{2x+6} \\ & 0 \leftarrow \text{remainder} \end{array}$$

$$\boxed{\begin{array}{r} x+3=0 \\ x=-3 \end{array}}$$

$$1x^2+5x+6$$

$$\begin{array}{r|rrr} -3 & 1 & 5 & 6 \\ + & \downarrow & -3 & -6 \\ \hline & 1 & 2 & 0 \leftarrow \text{r} \end{array}$$

$$1x+2$$

$$\boxed{\frac{\text{dividend}}{\text{divisor}} = \text{quotient}} \quad \boxed{\frac{P(x)}{x-a} = Q(x)}$$

$$x^2+5x+6=(x+2)(x+3) \quad \boxed{P(x)=Q(x)(x-a)}$$

$$\boxed{\text{dividend} = (\text{quotient})(\text{divisor})}$$

The remainder is the y value when $x=-3\ (-3,0).\ y=0, r=0.$

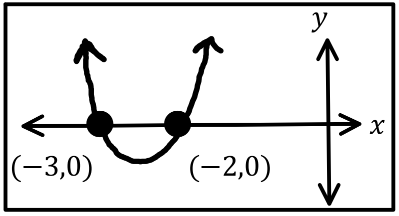
Factor Theorem :

$\boxed{\text{Sub the Root of the Divisor (Opposite*)}}$

$f(x)=x^2+5x+6$
$f(-3)=(-3)^2+5(-3)+6$
$f(-3)=0 \leftarrow$ Remainder
$(-3,0)$ $\boxed{x+3 \text{ Is a factor}}$

$$\boxed{\begin{array}{rr} x+3=0 & \boxed{\begin{array}{r} x-a=0 \\ x=a \end{array}} \\ x=-3 & \\ \boxed{a=-3} & \begin{array}{r} f(a)=0 \\ f(-3)=0 \end{array} \end{array}}$$

x	y
-3	0



$$\frac{x^2+5x+9}{x+3} = x+2+\frac{3}{x+3}$$

$$\begin{array}{rl} & x+2 \\ x+3 & \overline{)\,x^2+5x+9} \\ - & \underline{x^2+3x} \\ & 2x+9 \\ - & \underline{2x+6} \\ & 3 \leftarrow \text{r} \end{array}$$

$$1x^2+5x+9$$

$$\begin{array}{r|rrr} -3 & 1 & 5 & 9 \\ + & \downarrow & -3 & -6 \\ \hline & 1 & 2 & 3 \leftarrow \text{r} \end{array}$$

$$1x+2$$

$$\boxed{\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}} \quad \boxed{\frac{P(x)}{x-a} = Q(x)+\frac{R}{x-a}}$$

$$x^2+5x+9=(x+2)(x+3)+3$$

The remainder is the y value when $x=-3\ (-3,3).\ y=3, r=3.$

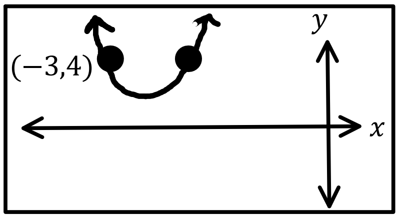
$$\boxed{\text{dividend} = (\text{quotient})(\text{divisor}) + \text{remainder}} \quad \boxed{P(x)=Q(x)(x-a)+R}$$

Remainder Theorem :

$f(x)=x^2+5x+10$
$f(-3)=(-3)^2+5(-3)+10$
$f(-3)=4 \leftarrow$ Remainder
$(-3,4)$ $\boxed{x+3 \text{ Is Not a factor}}$

$$\boxed{\begin{array}{rr} x+3=0 & \boxed{\begin{array}{r} x-a=0 \\ x=a \end{array}} \\ x=-3 & \\ \boxed{a=-3} & f(a)=R \\ \boxed{R=4} & f(-3)=4 \end{array}}$$

x	y
-3	4



$$\boxed{\begin{array}{lll} \textit{Factor Theorem}: & \text{If } (x-a) \text{ is a Factor of } f(x)\text{, then:} & f(a)=0 \\ \textit{Remainder Theorem}: & \text{If } (x-a) \text{ is Not a Factor of } f(x)\text{, then:} & f(a)=\textit{remainder} \end{array}}$$

$$\boxed{\begin{array}{rl} & 16 \\ 4 & \overline{)\,64} \\ - & \underline{4} \\ & 24 \\ - & \underline{24} \\ & 0 \end{array} \quad \begin{array}{c} \dfrac{64}{4}=16 \\ \\ 64=16\times 4 \end{array}} \boxed{\begin{array}{rl} & 16 \\ 4 & \overline{)\,65} \\ - & \underline{4} \\ & 25 \\ - & \underline{24} \\ & 1 \end{array} \quad \begin{array}{c} \dfrac{65}{4}=16+\dfrac{1}{4} \\ \\ 64=16\times 4+1 \end{array}}$$

C12 - 3.0 - Poly Notes

If Quartic do twice, If Quintic do thrice…

$$\frac{x^3+x^2-8x+6}{x-2} = x^2+3x-2+\frac{3}{x-2}$$

$$x^3+x^2-8x+6 = (x^2+3x-2)(x-2)+3$$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & 6 \\ + & \downarrow & 2 & 6 & -4 \\ \hline & 1 & 3 & -2 & 2 \end{array}$$

$1x^2+3x-2 \qquad R=2$

$$\begin{array}{rl} & x^2+3x-2 \\ x-2\,) & x^3+x^2-8x+6 \\ - & x^3-2x^2 \\ & +3x^2-8x \\ - & 3x^2-6x \\ & -2x+6 \\ - & -2x+4 \\ & 2 \end{array}$$

$R=2$

$f(x)=x^3+x^2-8x+6$
$f(2)=(2)^3+(2)^2-8(2)+6$
$f(2)=8+4-16+6$
$f(2)=2$ $\quad$ $x-2$ Is Not a Factor

$$\frac{x^3+2x-12}{x-2}$$

$1x^3+0x^2+2x-12$

The Gap!

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 2 & -12 \\ + & & & & \\ \hline \end{array}$$

Same Process*!

$x-2\,)\ x^3+0x^2+2x-12$

$$\frac{x^3+2x^2-6x-12}{x+2}$$

$$\begin{array}{rl} & x^2+0x-6 \\ x+2\,) & x^3+2x^2-6x-12 \end{array}$$

Same Process*! …

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -6 & -12 \\ + & \downarrow & -2 & 0 & 12 \\ \hline & 1 & 0 & -6 & 0 \end{array}$$

$1x^2+0x-6 \qquad R:0$

$x^2-6 \qquad R:0$

Factoring Cubics : Quartic, Quintic … Repeat

$y=x^3+2x^2-5x-6$

Solve by Inspection :

Potential Factors: $\pm 1,2,3,6$

Factors of Constant (#*).

$\boldsymbol{f(x)=3x^2+5x-2}$

$$\&\ \frac{Factors\ of\ Constant}{Factors\ of\ "a"} \quad \pm 2, \pm 1, \pm\frac{2}{3}, \pm\frac{1}{3}$$

$f(1)=(1)^3+2(1)^2-5(1)-6$
$f(1)=1+2-5-6$
$f(1)=-8$ $\quad$ $(x-1)$ is NOT a Factor
$f(-1)=(-1)^3+2(-1)^2-5(-1)-6$
$f(-1)=-1+2+5-6$
$f(-1)=0$ $\quad$ $(x+1)$ is a Factor

$6^3=216$ its not 6.

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ + & \downarrow & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$1x^2+1x-6 \qquad R=0$

$(x+3)(x-2)$

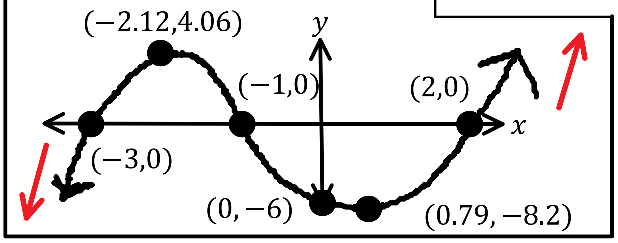
Store x : Repeat!
x^3+2x^2-5x-6
1 Store x Enter
TI84 Up Up Enter
TI83 2nd Entry (Twice*)

OR
Graph
$x-int$
2nd
Calc
Zero*

$$x^3+2x^2-5x-6=(x+3)(x-2)(x+1)$$

Graphing : Check on Calculator

$y=f(x)=x^3+2x^2-5x-6$



$x+3=0$ $\quad$ $x-2=0$ $\quad$ $x+1=0$
$x=-3$ $\quad$ $x=2$ $\quad$ $x=-1$

x	y
-3	0
-1	0
0	-6
2	0

$y-int\,; x=0$
$f(0)=(0)^3$ …
$f(0)=-6$

End Behavior : $+x^3$ … $\quad +x^{odd}$ $\quad Q3, Q1$ ↙ ↗

Domain : $x\in\mathbb{R}$ $\qquad$ Range : $y\in\mathbb{R}$

$f(x)>0$ Positive $\qquad$ $f(x)<0$ Negative

$-3<x<-1, x>2$ $\qquad$ $x<-3, -1<x<2$

Find Cubic Equation with $x-int$: $\left(0,-\frac{1}{2}\right)$
& (0,2) (Multiplicity of 2 and $y-int$ $(0,-8)$).

$y=a(x-\#)^{\#}(x-\#)^{\#}$ …
$y=a(2x+1)^1(x-2)^2$
$-8=a(2(0)+1)^1((0)-2)^2$
$-8=4a$
$a=-2$
$y=-2(2x+1)^1(x-2)^2$

$x=-\frac{1}{2}$
$2x+1=0$
$(0,8)$

$x=2$
$x-2=0$

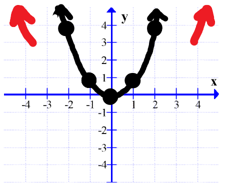
C12 - 3.0 - Poly Notes

Domain
$x \in \mathbb{R}$

End Behavior :
(Leading Term)

TOV

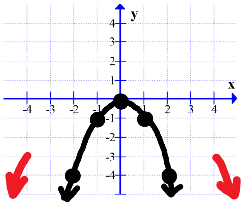
x	y
-10^*	$+/-?$
$+10^*$	$+/-?$



$y = x^2 \ldots$

$+\#x^{even}$

Q2,Q1



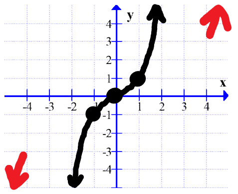
$y = -x^2 \ldots$

$-\#x^{even}$

Q3,Q4

Range

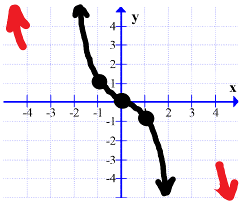
$y \geq \#$ $y \leq \#$	$y \in \mathbb{R}$



$y = x^3 \ldots$

$+\#x^{odd}$

Q3,Q1



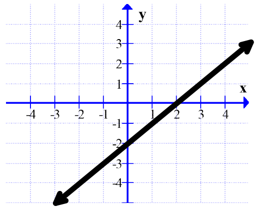
$y = -x^3 \ldots$

$-\#x^{odd}$

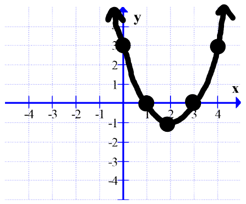
Q2,Q4

Multiplicity : Behavior near $x - intercept.$

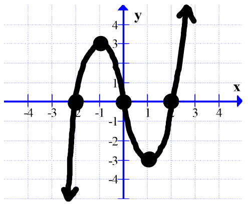
$y = (x-2)^1$



$y = (x-1)^1(x-3)^1$

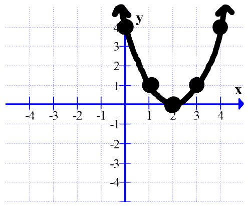


$y = x^1(x-2)^1(x+2)^1$



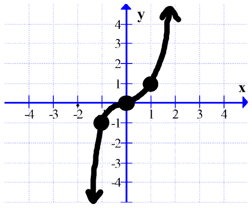
Degree 1: Straight Through

$y = (x-2)^2$



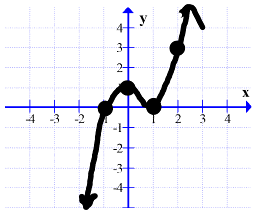
Degree 2: Bounce off

$y = x^3$

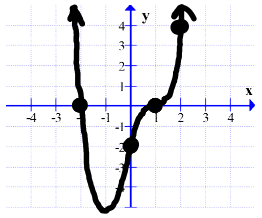


Degree 3: Chair Shape

$y = (x+1)^1(x-1)^2$

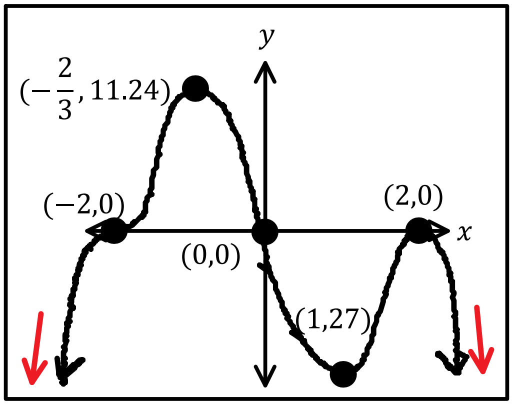


$y = (x+2)^1(x-1)^3$



Combinations

Graphing : $y = -x^1(2-x)^2(x+2)^3$



$-x^6 = -x^{even}$ ↓↓ End Behavior :

$x = 0$

$(0,0)$

$2 - x = 0$

$x = 2$

$(2,0)$

$x + 2 = 0$

$x = -2$

$(-2,0)$

$x - int:$

Straight Through	Bounce Off	Chair Shape	Multiplicity :

$y = -x(2-x)^2(x+2)^3$

$y = -(0)(2-(0))^2((0)+2)^3$

$y = 0$

$(0,0)$

$y - int\,; x = 0$

x	y
0	0

Long/Synthetic Division
Factor/Remainder Theorem
Substitution/TOV/Graph
Division/Multiplication Form
The Gap/Potential Factors
Solve by Inspection/Intercepts
Domain & Range/Pos/Neg
End Behavior/Multiplicity
Fine Equation/Word Problems
Find k

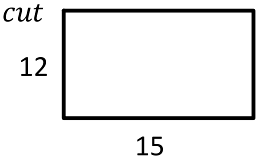
$y = x^2 + x - 1$
Quadform
$x = 0.618, x = -1.61$
OR 2nd Calc Zero

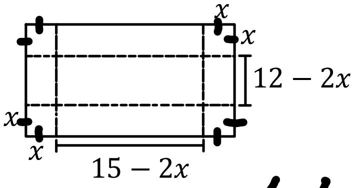
Use Long Division if Divisors :
-Binomials Coefficient≠ 1(ie. $2x + 1$)
-Trinomials/Quadratic/etc. (ie. $x^2 - 3$)

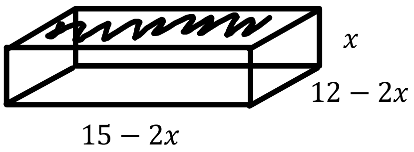
C12 - 3.0 - Poly Notes

An open rectangular box is made by cutting equal integer lengths from each corner of a 12 cm by 15 cm rectangular piece of cardboard, then folding up the sides. Find the length of the square that must be cut from each corner so the box has a volume of 162 cm^3. And find length to cut for Max Volume and find Max Volume.

$let\ x = length\ to\ cut$







$V = lwh$

$V = (12-2x)(15-2x)x$

$162 = (12-2x)(15-2x)(x)$

$162 = 180x - 54x^2 + 4x^3$

$0 = 4x^3 - 54x^2 + 180x - 162$

$0 = 2x^3 - 27x^2 + 90x - 81$

Potential Factors: of 81: $\cancel{\pm 27}, \cancel{\pm 9}, +3, +1$

Solve by inspection : $Check: x = 3,1$

$f(x) = 2x^3 - 27x^2 + 90x - 81$

$f(3) = 2(3)^3 - 27(3)^2 + 90(3) - 81$

$f(3) = 54 - 243 + 270 - 81$ $= 0$ $(x-3)$is a Factor

$Domain:$

$x > 0$ x cant be negative!

$x < 6$ Cant cut 2 6′s off a 12!

$$\begin{array}{c|cccc} 3 & 2 & -27 & 90 & -81 \\ + & \downarrow & 6 & -63 & 81 \\ \hline & 2 & -21 & 27 & 0 \end{array}$$

$2x^2 - 21x + 27$

$(2x-3)(x-9)$

$\cancel{x = 1.5}$ $\cancel{x = 9}$

Reject non-integers

$D: x < 6$

$\therefore x = 3\ cm$

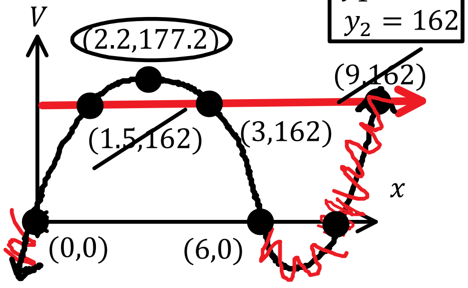
Maximum Volume :

$\boxed{\text{2nd}}\ \boxed{\text{Calc}}\ \boxed{\text{Max}}$

$V = (12-2x)(15-2x)x$

$y_1 = V$

$y_2 = 162$



Check Answer :

$l = 15 - 2x$ $w = 12 - 2x$ $h = x$ $V = lwh$

$l = 15 - 2(3)$ $w = 12 - 2(3)$ $h = 3$ $V = 9 \times 6 \times 3$

$l = 9$ $w = 6$ $V = 162\ cm^2$

Factor/Remainder Theorem : $x + 3 = 0$

$x = -3$

Find k if $(x+3)$ is a factor of f(x).

$f(x) = x^3 + 2x^2 + kx - 6$ $\boxed{f(-3) = 0}$

$f(-3) = (-3)^3 + 2(-3)^2 + k(-3) - 6 = 0$

$-15 - 3k = 0$

$f(x) = x^3 + 2x^2 - 5x - 6$ $k = -5$

$(-3,0)$ Check on Calc ✓

Find k if $f(x)$ is divided by $(x-1)$ and the remainder is -8.

$f(x) = x^3 + 2x^2 - 5x + k$ $\boxed{f(1) = -8}$

$f(x) = (1)^3 + 2(1)^2 - 5(1) + k = -8$

$-2 + k = -8$ $x - 1 = 0$

$f(x) = x^3 + 2x^2 - 5x - 6$ $k = -6$ $x = 1$

Find k and the remainder when f(x) is divided by $(x+1)$ if $(x-2)$ is a Factor. $\boxed{\text{Then Fully Factor...}}$

$f(x) = x^3 - 6x^2 + 11x + k$

$f(2) = (2)^3 - 6(2)^2 + 11(2) + k = 0$

$k = -6$

$f(x) = x^3 - 6x^2 + 11x - 6$

$f(-1) = (-1)^3 - 6(-1)^2 + 11(-1) - 30 = r$

$r = -59$

Find k & m if $f(x) = x^3 + 2x^2 + kx + m$ is divided by $(x+3)$ and $(x+1)$ the remainder is the same $= -2$.

$f(-1) = (-1)^3 + 2(-1)^2 + k(-1) + m = -2 = r$

$1 - k + m = -2 = r$

$f(-3) = (-3)^3 + 2(-3)^2 + k(-3) + m = -2 = r$

$-9 - 3k + m = -2 = r$

$\rightarrow r = r \leftarrow$

$1 - k + m = -9 - 3k + m$ $\boxed{\text{OR}}$ Eliminate!

$k = -5$

$\downarrow$

$1 - k + m = -2 \leftarrow$

$1 - (-5) + m = -2$

$m = -8$

$f(x) = x^3 + 2x^2 - 5x - 8$

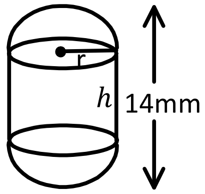
C12 - 3.0 - Poly Notes

Like Term : Same Letter(s), Same Exponent(s).
Coefficient : A number in front of (multiplying) a variable
Polynomial : Terms with Variables with Whole Number Exponents*. (ie. 0,1,2,3…)
Monomial : One term. **Binomial :** Two terms. **Trinomial :** Three terms.
Polynomial : Any # of terms (Variables w/whole # exponents 0,1,2,3…)
Degree of Term : The Variable Exponent or Sum of Variable Exponents.
Leading Term : The Term with the Highest Degree.
Leading Coefficient : Coefficient of Highest Degree Term.
Degree of Polynomial : Degree of Leading term.
Constant : A number.

Polynomial
$5x - 2x^3, x^2 - 1, x + 1, 5, |x|$

Not Polynomial
$x^{-2}, x^{\pi}, 2^x, \frac{1}{x}, \sqrt{x} = x^{\frac{1}{2}}, logx, sinx$

$V = 108\pi$



$h = 14 - 2r$

$h = 14 - 2(3)$
$h = 8$

$$V = \pi r^2 h + \frac{4\pi r^3}{3}$$
$$108\pi = \pi r^2(14 - 2r) + \frac{4\pi r^3}{3}$$
…
$$0 = 2r^3 - 57r^2 - 108$$
…
$$\boxed{r = 3\ cm}$$

$$V = \pi r^2 h + \frac{4\pi r^3}{3}$$
$$108\pi = \pi(3)^2(8) + \frac{4\pi(3)^3}{3}$$
$$339.3 = 339.3\ mm^3$$
✓

Find three consecutive odd integers whose product is −105.

$Let\ x = 1st\ \#$
$Let\ x + 2 = 2nd\ \#$
$Let\ x + 4 = 3rd\ \#$

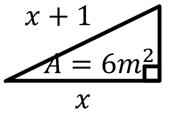
$$x(x + 2)(x + 4) = -105$$
…
$$x^3 + 6x^2 + 8x + 105 = 0$$
$$(-7)^3 + 6(-7)^2 + 8(-7) + 105 = 0$$
$$f(-7) = 0 \qquad (x + 7)\ is\ a\ factor$$

$$\begin{array}{r|rrrr} -7 & 1 & 6 & 8 & 105 \\ + & & -7 & 7 & -105 \\ \hline & 1 & -1 & 15 & 0 \end{array}$$

$x^2 - x - 15$
Quad
$x = 4.4,$

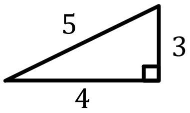
$$\boxed{\begin{array}{l} 1st\ \# = -7 \\ 2nd\ \# = -5 \\ 3rd\ \# = -3 \end{array}}$$

$(-3)(-5)(-7) = -105$ ✓



$a^2 + b^2 = c^2$
$x^2 + b^2 = (x + 1)^2$
…
$$\boxed{b = \sqrt{2x + 1}}$$

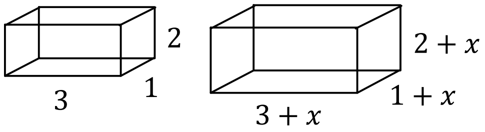
$$A = \frac{bh}{2}$$
$$6 = \frac{(x)(\sqrt{2x + 1})}{2}$$
$$2 = x(\sqrt{2x + 1})$$
…
$$144 = x^2(2x + 1)$$
$$0 = 2x^3 + x^2 - 144$$
…
$$\boxed{x = 4\ m}$$



✓ $3^2 + 4^2 = 5^2 \qquad 6 = \frac{3 \times 4}{2}$

Rectangular prism 3x2x1, all sides increased by same amount to increase volume ten times.

$let\ x = increase$



$V = lwh$
$60 = (3 + x)(1 + x)(2 + x)$
$60 = (3 + 4x + x^2)(2 + x)$
$0 = x^3 + 4x^2 + 3x + 2x^2 + 8x + 6 - 60$
$0 = x^3 + 6x^2 + 11x - 54$

$V = lwh$
$V = 3(1)(2)$
$\boxed{V = 6}$
$6 \times 10 = \boxed{60}$

$f(2) = 0$
$\boxed{x = 2}$

$$\begin{array}{r|rrrr} 2 & 1 & 6 & 11 & -54 \\ + & & 2 & 16 & 54 \\ \hline & 1 & 8 & 27 & 0 \end{array}$$

$V = lwh$
$V = 5(3)(4)$
$V = 60$

$x^2 + 8x + 27$
No Solution

Break Even Point? $Revenue(x) = Cost(x)$

$$R(x) = 100x - x^2$$
$$C(x) = \frac{1}{3}x^3 - 6x^2 + 89x + 100$$
$$100x - x^2 = \frac{1}{3}x^3 - 6x^2 + 89x + 100$$
…
$$0 = x^3 - 15x^2 - 33x + 300$$
$$\ldots \boxed{x = 3.92, 15.88,} \cancel{-4.81}$$