

# C12 - 6.4 - Proofs Conjugate Notes

Conjugate:

$$a + b \longleftrightarrow a - b$$

$$a - b \longleftrightarrow a + b$$

Conjugate:

$$1 - \sin x \longleftrightarrow 1 + \sin x$$

$$1 + \sin x \longleftrightarrow 1 - \sin x$$

Conjugate:

$$1 + \cos x \longleftrightarrow 1 - \cos x$$

$$1 - \cos x \longleftrightarrow 1 + \cos x$$

$$\frac{\square}{1 + \cos x} \times \frac{1 - \cos x}{1 - \cos x}$$

$$\frac{\square}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x}$$

$$\frac{\square}{1 - \cos x} \times \frac{1 + \cos x}{1 + \cos x}$$

$$\frac{\square}{1 - \sin x} \times \frac{1 + \sin x}{1 + \sin x}$$

Prove that the two sides are equal.

$$\frac{\sin x}{1 + \cos x}$$

$$\frac{1 - \cos x}{\sin x}$$

The conjugate

$$\times \frac{1 - \cos x}{1 - \cos x}$$

$$\frac{\sin x}{1 + \cos x} \times \frac{1 - \cos x}{1 - \cos x}$$

$$\frac{(1 - \cos x)}{\sin x}$$

- 1) Multiply the top and bottom by the conjugate of the denominator
- 2) FOIL the bottom
- 3) Pythagorean Identity
- 4) Simplify

$$\frac{\sin x (1 - \cos x)}{(1 + \cos x)(1 - \cos x)}$$

$$\frac{\sin x (1 - \cos x)}{1 - \cancel{\cos x} + \cancel{\cos x} - \cos^2 x}$$

$$\frac{(1 + \cos x)(1 - \cos x)}{1 - \cancel{\cos x} + \cancel{\cos x} - \cos^2 x} \quad \frac{(a + b)(a - b)}{a^2 - \cancel{ab} + \cancel{ab} + b^2}$$

FOIL (FL)  $a^2 - b^2$

$$\frac{\sin x (1 - \cos x)}{1 - \cos^2 x}$$

$$\sin^2 x + \cancel{\cos^2 x} = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\frac{\sin x (1 - \cos x)}{\sin^2 x}$$

Now we have the Pythagorean identity

$$\frac{(1 - \cos x)}{\sin x}$$

RHS ✓

Conj  
FL  
Pythag  
Simp

# C12 - 6.4 - Proofs Foil Conjugate Fact Frac Notes

$\frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$	$\text{Foil}$ $\frac{(\sin x - 1)(\sin x + 1)}{\sin^2 x - 1} = \frac{-\cos^2 x}{-\cos^2 x}$
$\frac{(1 - \cos x)}{\sin x}$ <div style="text-align: center; font-size: 2em; margin-top: 20px;">✓</div>	<div style="border: 1px solid black; display: inline-block; padding: 2px 5px; margin-bottom: 10px;">Conjugate!</div> $\frac{\sin x}{1 + \cos x} \times \frac{1 - \cos x}{1 - \cos x}$ $\frac{\sin x(1 - \cos x)}{1 - \cos^2 x}$ $\frac{\sin x(1 - \cos x)}{\sin^2 x}$ $\frac{(1 - \cos x)}{\sin x}$

$\frac{1 + \cos x}{\sin^2 x} = \frac{1}{1 - \cos x}$	$\text{Factor}$
$\frac{1 + \cos x}{1 - \cos^2 x}$ $\frac{1 + \cos x}{(1 - \cos x)(1 + \cos x)}$ $\frac{1}{1 - \cos x}$	<div style="text-align: center; font-size: 2em; margin-bottom: 20px;">✓</div> $\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} = 2 \csc^2 x$ <hr style="border: 0.5px solid black;"/> $\frac{1 - \cos x}{1 - \cos x} \times \frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} \times \frac{1 + \cos x}{1 + \cos x}$ $\frac{(1 - \cos x) + (1 + \cos x)}{(1 - \cos x)(1 + \cos x)}$ $\frac{1 - \cos x + 1 + \cos x}{1 - \cos^2 x}$ $\frac{2}{\sin^2 x}$ <div style="text-align: center; font-size: 2em; margin-top: 20px;">✓</div>