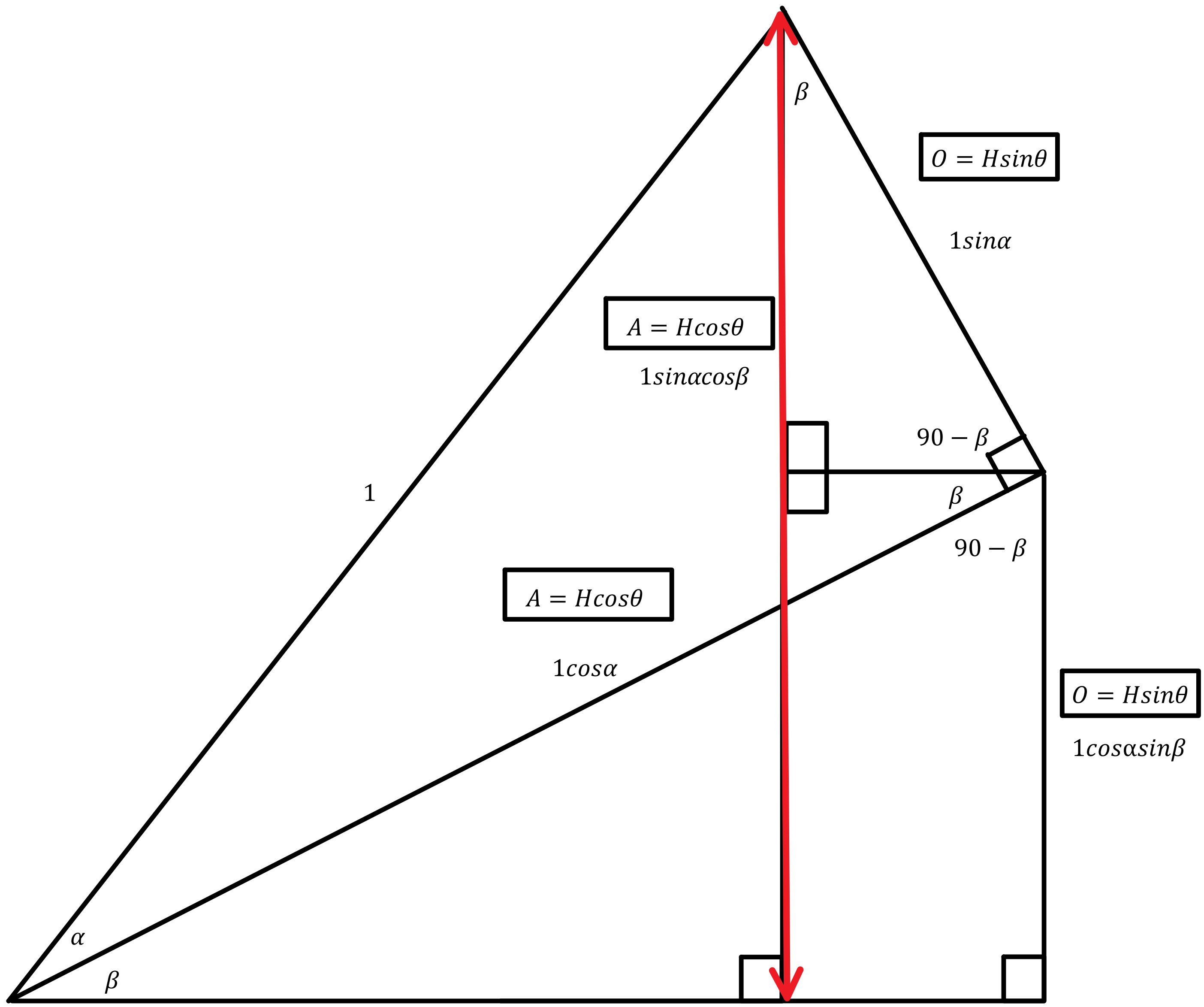
C12 - 6.5 - Sum and Differences Angle Theory



$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

C12 - 6.5 - Simplify/Expand Sum Difference Notes

$$tan\theta = \frac{sin\theta}{cos\theta}$$

$$\sin(a+b) = sinacosb + sinbcosa$$

$$\sin(x+\pi) = sinxcos\pi + sin\pi cosx$$
$$= sinx \times -1 + 0 \times cosx$$
$$= -sinx$$

$$\cos(a+b) = cosacosb - sinasinb$$

$$cos45cos30 + sin45sin30 = \cos(45^o - 30^o)$$
$$= cos15$$

$$\sin\left(\frac{\pi}{12}\right) =$$
$$sin15^0 =$$
$$\sin(45^0 - 30^0) = sin45cos30 - sin30cos45$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Rationalize!

$$\frac{\pi}{12} = 15^0$$

$15 = 45 - 30$ Or $\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{4} - \frac{\pi}{3}$

$$\sin(a-b) = sinacosb - sinbcosa$$

Sin is the same sign sincos:cossin
Cos is the opposite sign coscos:sinsin

$$\cos(-75) = \cos(-45-30)$$
$$= \cos(-45)\cos(30) + \sin(-45)\cos(30)$$

$$\cos(-x) = cosx \qquad \sin(-x) = -sinx$$

OR

$$cos75^o =$$
$$\cos(45^o + 30^o) = cos45cos30 - sin45sin30$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$
$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\cos(a-b) = cosacosb + sinasinb$$

$$sec15^o =$$
$$\frac{1}{cos15^o} =$$
$$\frac{1}{\cos(45^o - 30^o)} = \frac{1}{(cos45cos30 + sin45sin30)}$$
$$= \frac{1}{\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}}$$
$$= \frac{1}{\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)}$$
$$= \frac{1}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$
$$= 1 \times \frac{2\sqrt{2}}{\sqrt{3}+1}$$
$$= \frac{2\sqrt{2}}{\sqrt{3}+1}$$

$$\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right) + \sin\left(\frac{\pi}{6}+x\right)\sin\left(\frac{\pi}{6}-x\right) =$$
$$\cos\left(\frac{\pi}{6} + x - (\frac{\pi}{6} - x)\right) = \cos(2x)$$

$$sin255 = -sin105$$
$$\sin255 = \sin(360-105) = \sin 360cos105 - sin105cos360 = 0 - sin105(1) = -sin105 = -\sin(45+60)$$

A combination of special	$255 = 180 + 75$	$285 = 180 + 105$	$195 = 180 + 15$
and quadrantal angles...^*	$255 = 180 + (45 + 30)$	$285 = 180 + (60 + 45)$	$195 = 90 + 105$