$\sin^2 x = sinx \times sinx = (\sin(x))^2 \neq \sin x^2$
$2sin30 \neq sin60$
$\cos(x+\pi) \neq cosx + cos\pi$

C12 - 6.0 - Trig Notes

Reciprocal Identities : (Exponents/Factoring)

$$\frac{sinx}{sinx} = 1$$

$$\frac{\cos^3 x}{\cos^2 x} = \cos x$$

$$cosxtanx = cosx \times \frac{sinx}{cosx} = sinx$$

$$secxsinx = \frac{1}{cosx} \times sinx = \frac{sinx}{cosx} = tanx$$

$$\frac{tanx}{sinx} = \frac{\left(\frac{sinx}{cosx}\right)}{\cancel{sinx}_{1}}$$
$$\frac{sinx}{cosx} \div \frac{sinx}{1}$$
$$\frac{sinx}{cosx} \times \frac{1}{sinx}$$
$$\frac{1}{cosx} = secx$$

$$\frac{cosx}{tanx} = \frac{cosx}{\left(\frac{sinx}{cosx}\right)}$$
$$cosx \div \frac{sinx}{cosx}$$
$$cosx \times \frac{cosx}{sinx} = \frac{\cos^2 x}{sinx}$$

Pythagorean Identities : (Fractions/LCD)

$$\frac{1}{sinx} - sinx$$
$$\frac{1}{sinx} - sinx \times \frac{sinx}{sinx}$$
$$\frac{1}{sinx} - \frac{\sin^2 x}{sinx}$$
$$\frac{1-\sin^2 x}{sinx}$$
$$\frac{\cos^2 x}{sinx}$$

$$\frac{\frac{1}{cosx} - cosx}{1 - \frac{sinx}{cosx}} \qquad LCD = cosx$$
$$\left(\frac{\frac{1}{cosx} - cosx}{1 - \frac{sinx}{cosx}}\right) \times \frac{cosx}{cosx}$$
$$\frac{1-\cos^2 x}{cosx - sinx}$$
$$\frac{\sin^2 x}{cosx - sinx}$$

Multiply the top and bottom by the LCD $= cos\theta$

$$\frac{1}{1+cosx} + \frac{1}{1-cosx}$$
$$\frac{1-cosx}{1-cosx} \times \frac{1}{1+cosx} + \frac{1}{1-cosx} \times \frac{1+cosx}{1+cosx}$$
$$\frac{(1-cosx)+(1+cosx)}{(1-cosx)(1+cosx)}$$
$$\frac{1-cosx+1+cosx}{1-\cos^2 x}$$
$$\frac{2}{\sin^2 x} = 2\csc^2 x$$

Sum and Difference Identities :

$$sin3xcosx + cos3xsinx = \sin(3x+x) = sin4x$$

$$\sin(x+\pi) = sinxcos\pi + sin\pi cosx$$
$$= sinx \times -1 + 0 \times cosx$$
$$= -sinx$$

$$cos45cos30 + sin45sin30 = \cos(45^o - 30^o)$$
$$= cos15^o \; \ldots \; \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\sin\left(\frac{\pi}{12}\right) = \qquad \boxed{\frac{\pi}{12} = 15^0}$$
$$sin15^0 =$$
$$\sin(45^0 - 30^0) = sin45cos30 - sin30cos45$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

Rationalize!

$$= \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$sec15^o =$$

Do $cos15^o$and flip it!

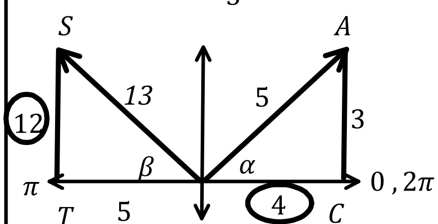
$$cos15^o \; \ldots \; \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\frac{1}{cos15^o} = \frac{2\sqrt{2}}{\sqrt{3}+1} = \frac{2\sqrt{6}+\sqrt{3}}{2}$$

$$\boxed{\cos(a-b) = cosacosb + sinasinb}$$

$$\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right) + \sin\left(\frac{\pi}{6}+x\right)\sin\left(\frac{\pi}{6}-x\right) =$$
$$\cos\left(\frac{\pi}{6}+x-\left(\frac{\pi}{6}-x\right)\right) =$$
$$= \cos(2x)$$

$$Solve: \quad sin\alpha = \frac{3}{5}\,; QI \qquad cos\beta = -\frac{5}{13}\,; QII$$



SOHCAHTOA

$a^2 + b^2 = c^2$
$a = 4$

$a^2 + b^2 = c^2$
$b = 12$

$$\sin(\alpha+\beta) = sin\alpha cos\beta + cos\alpha sin\beta$$
$$= \frac{3}{5} \times -\frac{5}{13} + \frac{4}{5} \times \frac{12}{13}$$
$$= -\frac{3}{13} + \frac{48}{65}$$
$$= \frac{33}{65}$$

$$sin2\alpha = 2sin\alpha cos\alpha$$
$$= 2 \times \frac{3}{5} \times \frac{4}{5}$$
$$= \frac{24}{25}$$

$$cos2\beta = 1 - 2\sin^2\beta$$
$$= 1 - 2\left(\frac{12}{13}\right)^2$$
$$= -\frac{119}{169}$$

C12 - 6.0 - Trig Notes

Simplify to $sinx \ or \ cosx$:

$1 - cos2x$	$1 + cos2x$
$1-(1-2\sin^2 x)$	$1+(2\cos^2 x - 1)$
$1-1+2\sin^2 x$	$1+2\cos^2 x - 1$
$2\sin^2 x$	$2\cos^2 x$

Choose the $cos2\theta$ to cross off the 1 (negative distribution*)

Double Angle Identities :

$\frac{1}{2}\sin 4x = 1\sin 2x\cos 2x$

$\boxed{1sin2x = 2sinxcosx}$

$2sin\pi = 4\sin\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\right) = 0$

Double the number in front.
Half the angle. Add a Cos

$8\sin 3x\cos 3x = 4\sin 6x$

Half the number in front.
Double the angle. Cos goes away

$4\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}\right) = 2\sin\left(\frac{\pi}{3}\right) = \frac{2\sqrt{3}}{2} = \sqrt{3}$

GCF

$2\cos^2 3x - 2\sin^2 3x =$
$2(\cos^2 3x - \sin^2 3x) = 2cos6x$

$cos4x = \cos^2 2x - \sin^2 2x$

Half the angle

$$\boxed{\begin{aligned} cos2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}}$$

Double the angle

$cos4x = 2\cos^2 2x - 1$

$4\cos^2 5 - 2 =$
$2(2\cos^2 5 - 1) = 2cos10$

$1 - 2\sin^2 2x = cos4x$

$1 - 2\sin^2 \pi = cos2\pi = 1$

Proofs :

$tanx$	$\frac{sin2x}{1+cos2x}$
$\frac{sinx}{cosx}$	$\frac{2sinxcosx}{1+(\square)}$ Hold Fast!
	$\frac{2sinxcosx}{1+(2\cos^2 x - 1)}$
	$\frac{2sinxcosx}{2\cos^2 x}$
	$\frac{2sinxcosx}{2\cos^2 x}$
	$\frac{sinx}{\cos x}$

IDs
Fractions
Factoring
Conjugate

$\frac{sin2\theta + cos\theta}{cot\theta}$	$1 - cos2\theta + sin\theta$
$\frac{2sin\theta cos\theta + cos\theta}{cot\theta}$	$1-(1-2\sin^2\theta) + sin\theta$
$\frac{cos\theta(2sin\theta+1)}{cot\theta}$	$1 - 1 + 2\sin^2\theta + sin\theta$
$\frac{cos\theta(2sin\theta+1)}{\frac{cos\theta}{sin\theta}}$	$2\sin^2\theta + sin\theta$
$cos\theta(2sin\theta+1) \times \frac{sin\theta}{cos\theta}$	
$2\sin^2\theta + sin\theta$	

Conjugate :

$\frac{1-cosx}{sinx}$	$\frac{sinx}{1+cosx}$
	$\frac{sinx}{1+cosx} \times \boxed{\frac{1-cosx}{1-cosx}}$
	$\frac{sinx(1-cosx)}{\sin^2 x}$
	$\frac{(1-cosx)}{sin\,x}$ Simplify

FOIL (FL)

$(a+b)(a-b)$
$a^2 - \cancel{ab} + \cancel{ab} + b^2$
$a^2 - b^2$

$(1+cosx)(1-cosx)$
$1 - \cancel{cosx} + \cancel{cosx} - \cos^2 x$
$1 - \cos^2 x$
$\sin^2 x$ Pythag

C12 - 6.0 - Trig Notes

$$\frac{\cos 2x}{\sin 2x+1} \quad \Bigg| \quad \frac{1-\tan x}{1+\tan x}$$

$2\sin\theta\cos\theta + 1$
$2\sin\theta\cos\theta + \sin^2\theta + \cos^2\theta$
$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$
$(\sin\theta + \cos\theta)(\sin\theta + \cos\theta)$

$m^2 + 2mn + n^2$
$(m+n)(m+n)$

Factoring

$$\frac{\square}{2\sin\theta\cos\theta + 1}$$

$$\frac{\cos^2\theta - \sin^2\theta}{(\sin\theta + \cos\theta)(\sin\theta + \cos\theta)}$$

$$\frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\sin\theta + \cos\theta)(\sin\theta + \cos\theta)}$$

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$\frac{1-\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin\theta}{\cos\theta}}$$

Multiply the top and bottom by the LCD $= \cos\theta$

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

OR Conjugate This!

QED

$$\sin 3x \quad \Big| \quad 3\sin x - 4\sin^3 x$$

$$\sin(2x + x)$$
$$\sin 2x\cos x + \cos 2x\sin x$$
$$2\sin x\cos^2 x + (1 - 2\sin^2 x)\sin x$$
$$2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$$
$$2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$$
$$3\sin x - 4\sin^3 x$$

QED

OR Add fractions on top/bottom and flip/multiply.

$$\frac{1-\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin\theta}{\cos\theta}}$$

$$1-\frac{\sin\theta}{\cos\theta} \qquad \frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta} \qquad \frac{\cos\theta-\sin\theta}{\cos\theta}$$

$$1+\frac{\sin\theta}{\cos\theta} \qquad \frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta} \qquad \frac{\cos\theta+\sin\theta}{\cos\theta}$$

$$\frac{\frac{\cos\theta - \sin\theta}{\cos\theta}}{\frac{\cos\theta + \sin\theta}{\cos\theta}}$$

$$\frac{\cos\theta - \sin\theta}{\cos\theta} \times \frac{\cos\theta}{\cos\theta + \sin\theta}$$

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

$$\cos(x+y)\cos(x-y) = \cos^2 x - \sin^2 y$$
$$(\cos x\cos y - \sin x\sin y)(\cos x\cos y + \sin x\sin y) =$$
$$\cos^2 x\cos^2 y - \sin^2 x\sin^2 y =$$
$$\cos^2 x(1 - \sin^2 y) - (1 - \cos^2 x)\sin^2 y =$$
$$\cos^2 x - \cos^2 x\sin^2 y - \sin^2 y + \cos^2 x\sin^2 y =$$
$$\cos^2 x - \sin^2 y = \cos^2 x - \sin^2 y$$

$$\sec^2 x + \csc^2 x = (\tan x + \cot x)^2$$
$$= \tan^2 x + 2\tan x\cot x + \cot^2 x$$
$$1 + \tan^2 x + 1 + \cot^2 x = \tan^2 x + 2\frac{\sin x}{\cos x}\frac{\cos x}{\sin x} + \cot^2 x$$
$$2 + \tan^2 x + \cot^2 x = \tan^2 x + 2 + \cot^2 x$$

$$\frac{1}{1 - \sin(90 - \theta)} = \csc^2\theta + \cot\theta\csc\theta$$
$$\frac{1}{1 - (\cos\theta\sin 90^0 - \sin\theta\cos 90^0)} = \frac{1}{\sin^2\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{1}{\sin\theta}$$
$$\boxed{\frac{1+\cos\theta\times}{1+\cos\theta\times}}\frac{1}{1-\cos\theta} = \frac{1}{\sin^2\theta} + \frac{\cos\theta}{\sin^2\theta}$$
$$\frac{1+\cos\theta}{1-\cos^2\theta} =$$
$$\frac{1+\cos\theta}{\sin^2\theta} = \quad QED$$