C12 - 8.1 - $\log_b a = ?$ Definition Notes

The Definition of a Logarithm:

$\log_3 9 = ?$
$\log_3 9 = 2$
$? = 2$

Think: What power do you have to raise 3 to, to equal 9?

$\log_2 8 = ?$
$\log_2 8 = 3$
$? = 3$

8 equals 2 to what power?

$$\log_b a = c$$

'a' is the thing you are logging

'c' is the answer/exponent

'b' is the base.

Switching from Log Form to Exponential Form:

$$\log_b a = c$$ *Log Form*

$$a = b^c$$ *Exponential Form*

The thing you are Logging equals the Base to the other side.

Remember:
The base of the log is the base of the exponent.

The exponent is the Answer.

Log Form

$\log_2 16 = ?$
$\log_2 16 = 4$
$? = 4$

16 equals 2 to what power?

Exponential Form

$16 = 2^?$
$2^4 = 2^x$
$? = 4$

Log Form -> Exponential Form and Solve for x

$\log_2 16 = x$ — Set Log arbitrarily $= x$

$16 = 2^x$
$2^4 = 2^x$ — Exponential Form, Change of Base

$x = 4$ — Same Base: Make exponents equal to each other

$\log_2 16 = 4$

$\log_{\frac{1}{2}} 16 = x$

$16 = \left(\frac{1}{2}\right)^x$
$2^4 = (2^{-1})^x$
$2^4 = 2^{-x}$
$4 = -x$

Exponential Form
Change of Base
Exponent Laws
Solve

$x = -4$

$\log_3\left(\frac{1}{27}\right) = x$

$\frac{1}{27} = 3^x$ — Exponential Form

$\frac{1}{3^3} = 3^x$
$3^{-3} = 3^x$ — Change of Base, Exponent Laws

$x = -3$

$\log_{2a} 16a^4 = x$

$16a^4 = (2a)^x$
$(2a)^4 = (2a)^x$ — Exponential Form, Change of Base

$x = 4$

C12 - 8.1 - $\log_b x = c, \log_x a = c, \log_b a = x$ *Notes*

Log Form -> Exponential Form and Solve for x

The base of the log is the base of the exponent

$\log_5 125 = x$

$125 = 5^x$ Exponential Form

$5^3 = 5^x$ Change of Base

$x = 3$ Same Base: Make exponents equal to each other

$\log_4 x = 3$

$x = 4^3$ Exponential Form

$x = 64$ Solve

$\log_6 x = 2$

$x = 6^2$

$x = 36$

$\log_5 x = -2$

$x = 5^{-2}$

$x = \frac{1}{5^2}$ Exponent Laws

$x = \frac{1}{25}$ Solve

$\log_9 x = \frac{1}{2}$

$x = 9^{\frac{1}{2}}$

$x = \sqrt{9}$

$x = 3$

$\log_x 64 = 3$

$64 = x^3$ Exponential Form

$4^3 = x^3$ Change of Base

$x = 4$ Solve

$\log_x 32 = 5$

$32 = x^5$ Exponential Form

$2^5 = x^5$ Change of Base

$\sqrt[5]{2^5} = \sqrt[5]{x^5}$ Fifth Root Both Sides

$x = 2$ Solve

$\log_x 27 = \frac{3}{2}$

$27 = x^{\frac{3}{2}}$

$27^{\frac{2}{3}} = \left(x^{\frac{3}{2}}\right)^{\frac{2}{3}}$

$27^{\frac{2}{3}} = x^1$

$\sqrt[3]{27^2} = x$

$x = 9$

Take both/ sides to reciprocal exponent

$\log_2(x-5) = 3$

$x - 5 = 2^3$

$x = 8 + 5$

$x = 13$

$x - 5 > 0$

$x > 5$

$\log_{x-2} 1 = 2$

$1 = (x-2)^2$

$1 = (x-2)(x-2)$

$1 = x^2 - 4x + 4$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

~~$x = 3$~~ $x = 1$

$x - 2 > 0$

$x > 2$

$x - 2 \neq 1$

$x \neq 3$

$log_{x-3} 2 = 1$

$2 = (x-3)^1$

$2 = x - 3$

$x = 5$

$x - 3 > 0$

$x > 3$

$x - 3 \neq 1$

$x \neq 4$

$\log_2 16 = x + 2$

$16 = 2^{x+2}$

$2^4 = 2^{x+2}$

$4 = x + 2$

$x = 2$

OR

$\log_2 16 = x + 2$

$\log_2 16 - 2 = x$

$4 - 2 = x$

$x = 2$

Do Algebra First!

$\log_2 16 = x$

$16 = 2^x$

$2^4 = 2^x$

$x = 4$

$\log_2 16 = 4$