C12 - 8.0 - Logs Notes

Definition/Change Forms :

$\log_2 8 = 3$

$\log_2 16 = x$
$16 = 2^x$
$2^4 = 2^x$
$x = 4$

$\log_{\frac{1}{2}} 16 = x$
$16 = \left(\frac{1}{2}\right)^x$
$2^4 = (2^{-1})^x$
$2^4 = 2^{-x}$
$4 = -x$
$x = -4$

$\log_3 \left(\frac{1}{27}\right) = x$
$\frac{1}{27} = 3^x$
$\frac{1}{3^3} = 3^x$
$3^{-3} = 3^x$
$x = -3$

$\log_{2a} 16a^4 = x$
$16a^4 = (2a)^x$
$(2a)^4 = (2a)^x$
$x = 4$

$\log_5 125 = x$
$125 = 5^x$
$5^3 = 5^x$
$x = 3$

$\log_b a = c$
$a = b^c$

$\log_4 x = 3$
$x = 4^3$
$x = 64$

$\log_6 x = 2$
$x = 6^2$
$x = 36$

$\log_5 x = -2$
$x = 5^{-2}$
$x = \frac{1}{5^2}$
$x = \frac{1}{25}$

$\log_x 64 = 3$
$64 = x^3$
$4^3 = x^3$
$x = 4$

$\log_x 32 = 5$
$32 = x^5$
$2^5 = x^5$
$\sqrt[5]{2^5} = \sqrt[5]{x^5}$
$x = 2$

$\log_9 x = \frac{1}{2}$
$x = 9^{\frac{1}{2}}$
$x = \sqrt{9}$
$x = 3$

$\log_x 27 = \frac{3}{2}$
$27 = x^{\frac{3}{2}}$
$27^{\frac{2}{3}} = x^1$
$\sqrt[3]{27^2} = x$
$x = 9$

Domain :

$\log_2(5 - x) = 3$
$5 - x = 2^3$
$x = 5 - 8$
$x = -3$

$y = a\log_{\#^*}(b(x - h)) + k$

Domain : Set what you are logging > 0 and solve.

$b(x - h) > 0$

$5 - x > 0$
$-x > -5$
$x < 5$

$\log_{x-2} 1 = 2$
$1 = (x - 2)^2$
$1 = (x - 2)(x - 2)$
$1 = x^2 - 4x + 4$
$x^2 - 4x + 3 = 0$
$(x - 3)(x - 1) = 0$
$x = 3$ $x = 1$

Domain : Set the base $\#^* > 0$ and $\neq 1$ and solve.

$x - 2 > 0$
$x > 2$

$x - 2 \neq 1$
$x \neq 3$

$\log_{10} 100^2 =$
$2\log_{10} 100 =$
$2 \times 2 = 4$

$\log x^m = m\log x$

Bring Exponents Down In Front

$\log x^2$
$2\log x$

$\log\left(\frac{1}{2}\right)$
$\log 2^{-1}$
$-1\log 2$
$-\log 2$

$3\log 4^2$
$2 \times 3\log 4$
$6\log 4$

OR

$3\log 4^2$
$\log 4^{2\times 3}$
$\log 4^6$
$6\log 4$

Exponents/Coefficients :

$\log_5 5^4 = x$
$5^4 = 5^x$
$x = 4$

$\log_5 625 = x$
$\log_5 5^4 = x$
$4\log_5 5 = x$
$4 \times 1 = x$
$x = 4$

$\log_5 625 = x$
$625 = 5^x$
$5^4 = 5^x$
$x = 4$

Distribute/Factor :

$\log 3^{x+2}$
$(x + 2)\log 3$
$x\log 3 + 2\log 3$

$3x\log 7 - x\log 2 =$
$x(3\log 7 - \log 2)$

$GCF = x$

Change of Base :

$\frac{\log 16}{\log 4} = \log_4 16 = 2$

$\frac{\log_2 16}{\log_2 4} = \log_4 16 = 2$

Choose the Base you want! The Base is Arbitrary

$\log_8 16 = \frac{\log_2 16}{\log_2 8} = \frac{4}{3}$

$\frac{1}{\log_8 2} =$
$\frac{1}{\left(\frac{\log 2}{\log 8}\right)} =$
$1 \times \frac{\log 8}{\log 2} = \frac{\log 8}{\log 2}$

$b^{\log_b x} = x$
$\log b^{\log_b x} = \log x$
$\log_b x \log(b) = \log x$
$\frac{\log_b x \log(b)}{\log b} = \frac{\log x}{\log b}$
$\log_b x = \frac{\log x}{\log b}$

So the 1st line is true!

$\log_b x = \log_b x$
$x = x$

Take the base and the log to any exponent you like* $\neq 0$!

$\log_3 9 + \log_9 2$
$\log_{(3)^2}(9)^2 + \log_9 2$
$\log_9 81 + \log_9 2$
$\log_9 81 \times 2$
$\log_9 162$

$\log_{\frac{1}{2}} 4 =$
$\log_{(\frac{1}{2})^{-1}} 4^{-1} =$
$\log_2 4^{-1} =$
$-1\log_2 4 = -2$

$\log_4 x$
$\log_{\sqrt{4}} \sqrt{x}$
$\log_2 x^{\frac{1}{2}}$
$\frac{1}{2}\log_2 x$

C12 - 8.0 - Logs Notes

Positives on top,
Negatives on Bottom
& Vice Versa

log _____

Addition/Subtraction :

$\log_2 4 + \log_2 8 =$ $\boxed{logA + \log B = logAB}$
$\log_2 4 \times 8 =$
$\log_2 32 = (5)$ Add-Multiply

$log1 + log5 + log7 =$ $\boxed{logA + \log B + \log C = logABC}$
$log1 \times 5 \times 7 = (\log 35)$

$\log_3 27 - \log_3 3 =$ $\boxed{LogA - LogB = \log\left(\frac{A}{B}\right)}$
$\log_3 \frac{27}{3} =$
$\log_3 9 = (2)$ Subtract-Divide

$\boxed{-logA + logB \quad logB - logA \quad (\log\left(\frac{B}{A}\right)) \text{ Rearrange}}$

$log4 + log20 - log10 =$
$log\frac{4 \times 20}{10} = (log\,8)$

$log5 - log2 + log10 =$
$log\frac{5 \times 10}{2} = (log25)$

$log5 - log2 - log10 =$
$log\frac{5}{2 \times 10} = (\log\frac{1}{4})$

$\boxed{+LogA + LogB - LogC = \log\left(\frac{AB}{C}\right)}$

$\boxed{Log\left(\frac{A}{BC}\right) = LogA - LogBC}$
$\boxed{Log\left(\frac{A}{BC}\right) = LogA - (LogB + LogC)}$
$\boxed{Log\left(\frac{A}{BC}\right) = LogA - LogB - LogC}$

$logxy^2 =$
$logx + logy^2 =$
$(logx + 2logy)$

$logx^2y^2 =$
$logx^2 + logy^2 =$
$2logx + 2logy$

$\log(xy)^2 =$
$2logxy =$
$2(logx + logy) =$
$2logx + 2logy$

$logx + \log x =$
$\log x \times x = (\log x^2)$

$\log 3 + \log(x+1) =$
$log3(x+1) = (\log(3x+3))$

$\log(x-2) + \log(x+1) =$
$\log(x-2)(x+1) = (\log(x^2 - x - 2))$

$\log x^3 - \log x^2 =$
$\log\frac{x^3}{x^2} = (logx)$

$\log(x^2-1) - \log(x+1) =$
$\log\frac{x^2-1}{x+1} =$
$\log\frac{(x+1)(x-1)}{(x+1)} = (\log(x-1))$

$2loga - \frac{1}{2}logb + 2log\sqrt{c}$ Bring Coefficients Up 1st!
$loga^2 - logb^{\frac{1}{2}} + \log\left(c^{\frac{1}{2}}\right)^2$
$\log$ _____
$(\log\frac{a^2c}{\sqrt{b}})$

Given: $log5 = m$ $log7 = n$ Solve in terms of m and n.

$log25 = log5^2$
$= 2log5$
$= 2m$

$log35 = log5 + log7$
$= m + n$

$log350 = log5 + log7 + log10$
$= m + n + 1$

$\log_5 7 = \frac{log7}{log5}$
$= \frac{n}{m}$

$log0.49 = \log\frac{49}{100}$
$= log49 - log100$
$= log7^2 - 2$
$= 2log7 - 2$
$= 2n - 2$

Given: $log4 = a$ $log6 = b$ Solve in terms of a and b:

$log2 =$
$log\sqrt{4} =$
$log4^{\frac{1}{2}} =$ OR
$\frac{1}{2}log4 =$
$\frac{1}{2}a = \frac{a}{2}$

$log4 = a$
$log2^2 = a$
$2log2 = a$
$log2 = (\frac{a}{2})$

$\log\left(\frac{1}{2}\right) =$
$log2^{-1} =$
$-log2 = (-a)$

$log3 =$
$\log\frac{6}{2} =$
$log6 - log2 =$
$(b - \frac{1}{2}a)$

$log0.4 =$
$\log\left(\frac{4}{10}\right) =$
$log4 - log10 =$
$(a - 1)$

C12 - 8.0 - Logs Notes

Calculator : TI83 TI84 : Math Alpha Math

$log8 = 0.9031$ $\quad log_4 7 = \frac{log7}{log4}$ 1.4037

Delog Both Sides :

$\log_5(x+1) = \log_5 7$

$x+1 = 7$

$\boxed{x = 6}$

$\log_2(x-2) + \log_2(x+1) = 2$

$\log_2(x-2)(x+1) = 2$

$\log_2(x^2 - x - 2) = 2$

$x^2 - x - 2 = 2^2$

$x^2 - x - 2 = 4$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\boxed{x = 3}$ $\quad \cancel{x = -2}$

OR

$\log_2(x-2) + \log_2(x+1) = 2$

$\log_2(x^2 - x - 2) = \log_2 4$

$x^2 - x - 2 = 4$

$x^2 - x - 6 = 0$

... (Turn a number into a log!)

$2 = \log_2 m$

$2^2 = m$

$m = 4$

$2 = \log_2 4$

$\log_2(x-2) + \log_2(x+1) = 2$

$\log_2(x-2) - 2 = -\log_2(x+1)$

...

$x - 2 > 0$ $\quad \boxed{x > 2}$

$x - 1 > 0$ $\quad \cancel{x > -1}$

$\log_3(x-11) - \log_3(x-3) = 2$

$\log_3 \frac{x-11}{x-3} = 2$

$\frac{x-11}{x-3} = 3^2$

$\frac{x-11}{x-3} = 9$

$x - 11 = 9(x-3)$

$x - 11 = 9x - 27$

$16 = 8x$

$\cancel{x = 2}$ $\quad \boxed{x > 3}$

$2\log_5 x + \log_5 x = 3$

$\log_5 x^2 + \log_5 x = 3$

$\log_5 x^2 \times x = 3$

$\log_5 x^3 = 3$

$x^3 = 5^3$

$\boxed{x = 5}$ $\quad \boxed{x > 0}$

Bring Coefficients Up 1st!

$let\ m = logx$

$(logx)^2 - logx^3 = 4$

$(logx)^2 - 3logx = 4$

$m^2 - 3m - 4 = 0$

$(m-4)(m+1) = 0$

$m = 4$ $\quad logx = 4$ $\quad \boxed{x = 10^4}$

$m = -1$ $\quad logx = -1$ $\quad \boxed{x = 10^{-1}}$

Log Both Sides :

$4 = 2^x$

$log4 = log2^x$

$log4 = xlog2$

$\frac{log4}{log2} = x$

$\log_2 4 = x$

$\boxed{x = 2}$

$3 = 5^x$

$log3 = log5^x$

$log3 = xlog5$

$\frac{log3}{log5} = x$

$\log_5 3 = x$

$\boxed{x = 0.6826}$

$5^{0.6828} = 3$

Algebra :

$3 = 2^x - 1$

$\boxed{4 = 2^x}$

$8 = 2 \times 2^x$

$\boxed{4 = 2^x}$

OR

$8 = 2 \times 2^x$

$log8 = \log(2 \times 2^x)$

$\boxed{log8 = log2 + log2^x}$

$4 = 7^{2x+1}$

$log4 = log7^{2x+1}$

Brackets!

$log4 = (2x+1)log7$

$log4 = 2xlog7 + \log 7$

$log4 - log7 = 2xlog7$

$\frac{log4 - log7}{2log7} = x$

$x = \frac{(log4 - log7)}{(2log7)}$

$\boxed{x = -0.14379}$

OR

$4 = 7^{2x+1}$

$\log_7 4 = 2x + 1$

$\log_7 4 - 1 = 2x$

$x = \frac{\log_7 4 - 1}{2}$

$\boxed{x = -0.14379}$

$2^{2x-5} = 9^{x+2}$

$log2^{2x-5} = log9^{x+2}$

$(2x-5)log2 = (x+2)log9$

$2xlog2 - 5log2 = xlog9 + 2log9$

$2xlog2 - xlog9 = 2log9 + 5log2$

$x(2log2 - log9) = 2log9 + 5log2$

$\boxed{x = \frac{2log9 + 5log2}{2log2 - log9}}$

$\boxed{\log_9 2^{2x-5} = x + 2}$

$\boxed{\log_2 9^{x+2} = 2x - 5}$

$6 \times 3^x = 14^{2x-5}$

$\log(6 \times 3^x) = log14^{2x-5}$

$log6 + log3^x = log14^{2x-5}$

$log6 + xlog3 = (2x-5)log14$

$log6 + xlog3 = 2xlog14 - 5log14$

$2xlog14 - xlog3 = log6 + 5log14$

$x(2log14 - log3) = log6 + 5log14$

$\boxed{x = \frac{log6 + 5log14}{2log14 - log3}}$

C12 - 8.0 - Word Problem Notes

How long to earn $1500 on $10000 at 10%/year?

$$F = P(1+r)^t$$
$$11500 = 10000(1+0.1)^t$$
$$\frac{11500}{10000} = 1.1^t$$
$$1.15 = 1.1^t$$
$$log1.15 = log1.1^t$$
$$log1.15 = tlog1.1$$
$$\frac{log1.15}{log1.1} = t$$
$$\log_{1.1} 1.15 = t$$
$$t = 1.47\ years$$

$$\begin{array}{r} 10000 \\ +1500 \\ \hline 11500 \end{array}$$

How long to triple your money at 10%/year?

$$F = P(1+r)^t$$
$$3 = 1(1+0.1)^t$$
$$3 = 1.1^t$$
$$\log_{1.1} 3 = t$$
$$t = 11.53\ years$$

$P = 1 \rightarrow F = 3$

How long to grow $10000 to $12000 compounded quarterly at 10%?

$$F = P\left(1+\frac{r}{n}\right)^{tn}$$
$$12000 = 10000\left(1+\frac{0.1}{4}\right)^{4t}$$
$$1.2 = 1.025^{4t}$$
$$\log_{1.025} 1.2 = 4t$$
$$\frac{\log_{1.025} 1.2}{4} = t$$
$$t = 1.85\ years$$

Find the half-life of a substance decaying to 20% of its original in 500 years?

$$F = P(r)^{\frac{t}{T}}$$
$$20 = 100\left(\frac{1}{2}\right)^{\frac{500}{T}}$$
$$0.2 = 0.5^{\frac{500}{T}}$$
$$\log_{0.5} 0.2 = \frac{500}{T}$$

Cross Multiply

$$T = \frac{500}{\log_{0.5} 0.2} = 500 \log_{0.2} 0.5$$
$$T = 215.34\ years$$

An earthquake of magnitude 8 is 250 times as intense as an earth quake of what magnitude?

$$I = 10^{b-s}$$
$$250 = 10^{8-s}$$
$$\log_{10} 250 = 8 - s$$
$$s = 5.6\ magnitude$$

How long to grow 1000 Bacteria to 5000 at a continuous growth rate of 0.05?

$$F = Pe^{kt}$$
$$5000 = 1000e^{0.05t}$$
$$5 = e^{0.05t}$$
$$\frac{\ln_e 5}{0.05} = t$$
$$t = 32.2\ldots$$

Find the number of compounding periods to grow $10000 to $16288.95 at 10% in 5 years.

$$F = P\left(1+\frac{r}{n}\right)^{tn}$$
$$2 = 1\left(1+\frac{0.1}{n}\right)^{5n}$$
$$n = 2\ ; Semi-annually$$

$y_1 = y_2$
Find Intersection

A substance has a half-life of 5 years. How long to be ten percent of its original?

$$F = P(r)^{\frac{t}{T}}$$
$$10 = 100\left(\frac{1}{2}\right)^{\frac{t}{5}}$$
$$0.1 = 0.5^{\frac{t}{5}}$$
$$\log_{0.5} 0.1 = \frac{t}{5}$$
$$t = 16.61\ years$$

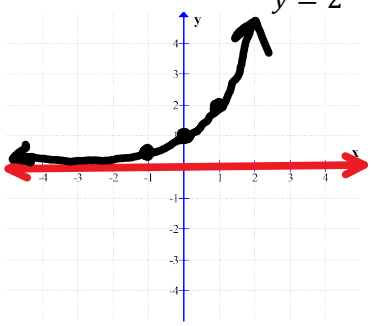
Definition
Change Forms
Solve
Domain
Exponents/Coefficients
Distribution/Factoring
Change of Basex2!
Addition/Subtraction
Log Both Sides/Delog
Turn a number into a log!
$let\ \ m = logx$
Word Problems
Graphing
Inverse

C12 - 8.0 - Log Graph/Inverse Notes

Graph Base^x
TOV
Switch x and y

Graph: $y = \log_2 x$

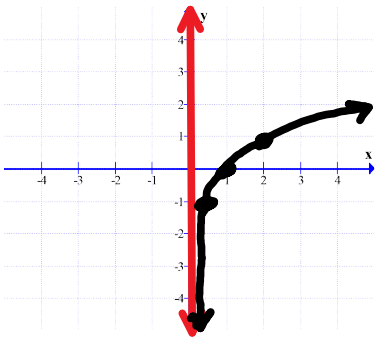
$y = 2^x$



x	y
-1	$\frac{1}{2}$
0	1
1	2

x	y
$\frac{1}{2}$	-1
1	0
2	1

$y = \log_2 x$



$HA: y = 0$

$x \in \mathrm{R}$

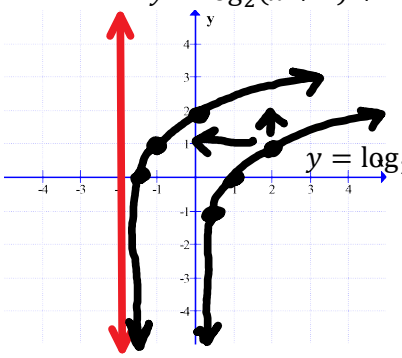
$y > 0$

$VA: x = 0$

$y \in \mathrm{R}$

$Domain:\ x > 0$

$y = \log_2(x+2) + 1$



$HT = -2$
$VT = +1$

$y = \log_2 x$

$x - int$

$y = \log_2(x+2) + 1$
$0 = \log_2(x+2) + 1$
$-1 = \log_2(x+2)$
$2^{-1} = x + 2$
$\frac{1}{2} = x + 2$
$x = -1.5$

$(-1.5, 0)$

$y - int$

$y = \log_2(x+2) + 1$
$y = \log_2(0+2) + 1$
$y = 2$

$(0,2)$

Domain

$x + 2 > 0$
$x > -2$

VA

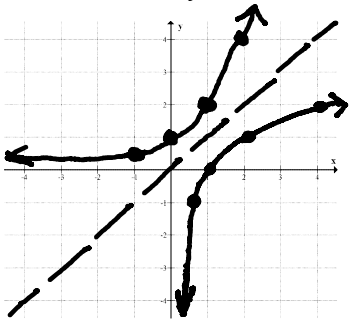
$x + 2 = 0$
$x = -2$

Inverse : *Switch x and y*

$y = 2^x$
$x = 2^y$
$\log x = \log 2^y$
$\log x = y \log 2$
$\frac{\log x}{\log 2} = y$
$\log_2 x = y$
$y = \log_2 x$
$f^{-1}(x) = \log_2 x$

$y = \log_2 x$
$x = \log_2 y$
$2^x = y$
$y = 2^x$
$f^{-1}(x) = 2^x$

$y = 2^x$



$y = \log_2 x$

$y = 2^{x+1} - 3$
$x = 2^{y+1} - 3$
$x + 3 = 2^{y+1}$
$\log(x+3) = (y+1)\log 2$
$\frac{\log(x+3)}{\log 2} = y + 1$
$\log_2(x+3) = y + 1$
$\log_2(x+3) - 1 = y$
$y = \log_2(x+3) - 1$
$f^{-1}(x) = \log_2(x+3) - 1$

$y = \log_2(x+3) - 1$
$x = \log_2(y+3) - 1$
$x + 1 = \log_2(y+3)$
$2^{x+1} = y + 3$
$2^{x+1} - 3 = y$
$y = 2^{x+1} - 3$
$f^{-1}(x) = 2^{x+1} - 3$