

LA - 1.6 - Inverse Matrices

Gauss - Jordan Method

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \rightarrow \left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array} \right] \leftarrow \text{Identity Matrix}$$

$$\left[\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{array} \right] \quad r_2 - 3r_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & -2 & -3 & 1 \end{array} \right] \quad r_1 + r_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{3}{2} & -\frac{1}{2} \end{array} \right] \quad r_2 \times -\frac{1}{2}$$

A^{-1}

Multiply the Inverse of a system's left hand side by the right hand size to solve the system.

$$AX = B \quad ; \quad X = A^{-1}B$$

May not have an inverse ie: $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Inverse Matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \quad A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} \quad \text{switch } a \text{ \& } d \quad \begin{bmatrix} d & b \\ c & a \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} \quad b \text{ \& } c \text{ change sign} \quad \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \quad \text{Divide by } D \quad \begin{bmatrix} -\frac{c}{D} & \frac{a}{D} \\ -\frac{c}{D} & \frac{a}{D} \end{bmatrix}$$

$D = ad - bc \neq 0$ $D = ad - bc$
 $D = \text{Determinant}$ $D = 1 \times 4 - 2 \times 3$
 $D = -2$

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \boxed{A \cdot A^{-1} = A^{-1} \cdot A = I}$$

$$\begin{bmatrix} 1 \times -2 + 2 \times \frac{3}{2} & 1 \times 1 + 2 \times -\frac{1}{2} \\ 3 \times -2 + 4 \times \frac{3}{2} & 3 \times 1 + 4 \times -\frac{1}{2} \end{bmatrix}$$