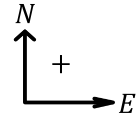
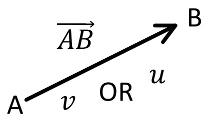
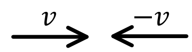


LA - 2.1 - Vectors

Vector
-Magnitude (Scalar)
-Direction



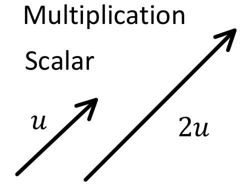
Adding Negative Vectors



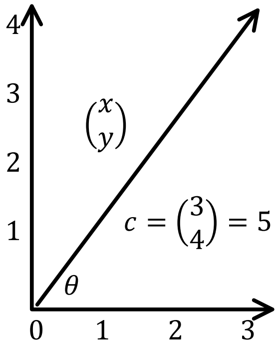
$u - v = u + (-v)$

Multiplication

Scalar



Distance/Angle of Vectors.



Find Distance.

$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$\sqrt{25} = \sqrt{c^2}$$
$$5 = c$$

Find Angle.

$$tan\theta = \frac{O}{A}$$
$$tan\theta = \frac{4}{3}$$
$$tan\theta = 1.333$$
$$\theta = \tan^{-1}(1.333)$$
$$\theta = 53.1^o$$

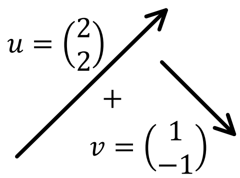
Table

x	y
+3	+4
+3	+4

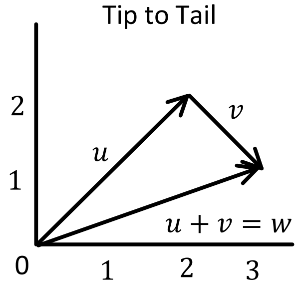
$c = 5, 53.1^o\ N\ o\ E$ Towards North From East

Vector Addition

$u + v$



Tip to Tail

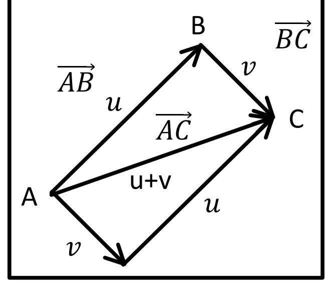


Table

x	y
+2	+2
+1	−1
+3	+1

$u + v = w = (3,1)$

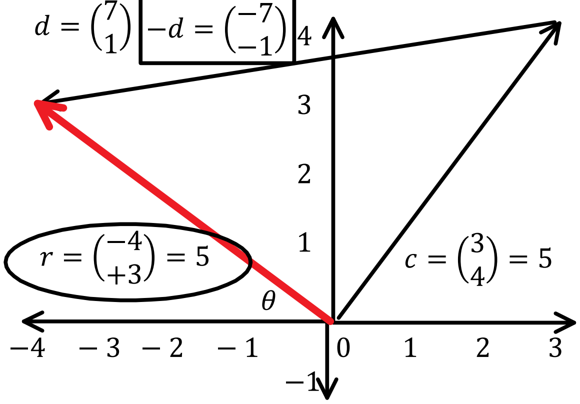
Parallelogram



Addition/Subtraction of Vectors

$c - d = c + (-d)$

$d = \binom{7}{1}$ $\quad -d = \binom{-7}{-1}$



x	y
+3	+4
−7	−1
−4	+3

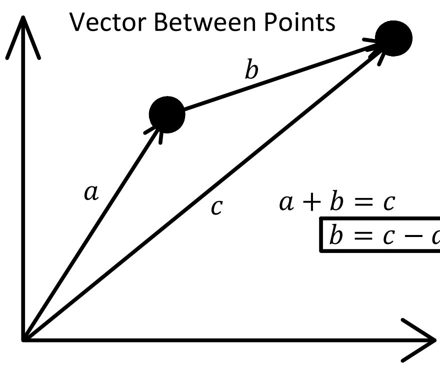
$c + d = r = (-4,3)$

$$x^2 + y^2 = r^2$$
$$(-4) + (3)^2 = r^2$$
$$16 + 9 = r^2$$
$$25 = r^2$$
$$\sqrt{25} = \sqrt{r^2}$$
$$5 = r$$

$$tan\theta = \frac{O}{A}$$
$$tan\theta = \frac{3}{4}$$
$$tan\theta = 0.75$$
$$\theta = \tan^{-1}(0.75)$$
$$\theta = 36.9^o$$

$c + d = r = 5, 36.9^o\ N\ o\ W$

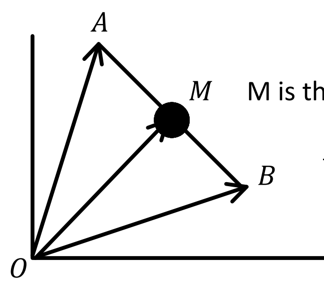
Vector Between Points



$a + b = c$

$b = c - a$

Midpoint



M is the Midpoint of $\overrightarrow{AB}$

$$\overrightarrow{OM} = \frac{1}{2}a + \frac{1}{2}b$$

LA - 2.2 - Vectors

Component Form

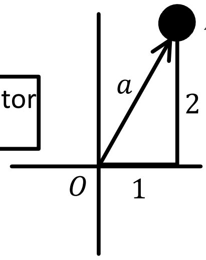
$\overrightarrow{OP} = \begin{pmatrix} x \\ y \end{pmatrix} = xi + yj = \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 1i + 2j$

Unit Vector Form

Unit Vectors

$i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad j = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

2 Dimensions



$P(1,2) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$|a|$: Modulus (Distance)

$|a| = \sqrt{x^2 + y^2}$

$|a| = \sqrt{1^2 + 2^2}$

$|a| = \sqrt{5}$

$-a = \begin{pmatrix} -a_1 \\ -a_2 \end{pmatrix}$

$-a = -1i - 2j \quad \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

$P(-1,-2)$

$ka = \begin{pmatrix} ka_1 \\ ka_2 \end{pmatrix}$; k *is a constant*

$a = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \quad a + b = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$

Vector (Dot) Product Multiplication

$a \bullet b = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \bullet \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = a_1b_1 + a_2b_2$

$a \bullet b = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \bullet \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3$

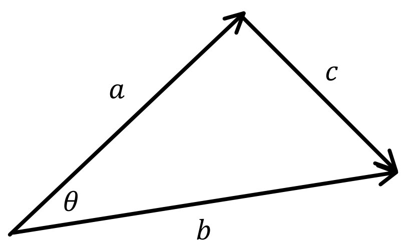
Properties

$a \bullet b = b \bullet a \qquad a \bullet a = |a|^2$

$a \bullet (b + c) = a \bullet b + a \bullet c$

$(a + b) \bullet (c + d) = a \bullet c + a \bullet d + b \bullet c + b \bullet d$

A measure of how closely two vectors align, in terms of the direction they point.



Angle Between Vectors

$a \bullet b = |a||b|cos\theta$ — From Cosine Law

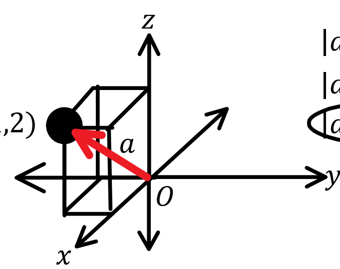
$a \bullet b = 0$ — Perpendicular ; $cos90 = 0$

$a \bullet b = |a||b|$ — Parallel ; $cos180 = 1$

3 Dimensions

$\overrightarrow{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = xi + yj + zk = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = 3i - 1j + 2k$

$P = (3,-1,2)$



$|a| = \sqrt{x^2 + y^2 + z^2}$

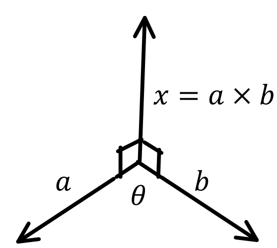
$|a| = \sqrt{3^2 + (-1)^2 + 2^2}$

$|a| = \sqrt{14}$

Unit Vectors

$i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad j = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad k = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Suppose $x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is perpendicular to both $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$



Vector Cross Product $a \times b$ (3 Dimensions*)

Area of a Rectangle with sides a & b.

$a \times b = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

$a \times b = -b \times a \qquad a \times a = 0$

$a \bullet (b \times c)$; Scalar Triple Product

$a \times (b + c) = a \times b + a \times c$

$(a + b) \times (c + d) = a \times b + a \times d + b \times c + b \times d$

$|a \times b| = |a||b|sin\theta$

$|a \times b| = \sqrt{(a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2 + (a_1b_2 - a_2b_1)^2}$