LA - 3.1 - Complex Numbers/Operations

$\boxed{\sqrt{-1} = i}$

This pattern repeats itself

$i^1 = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$
$i^5 = i$

Imaginary #'s

$i^2 = \left(\sqrt{-1}\right)^2 = -1$
$i^3 = \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} = -i$
$i^4 = \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} = 1$
$i^5 = \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} \times \sqrt{-1} = i$

$i^{-3} = \frac{1}{i^3} = \frac{1}{-1} = -1$

$\sqrt{-9} =$
$\sqrt{-9(-1)}$
$\sqrt{9} \times \sqrt{-1} =$
$3i$

Calculator
MODE a+bi

$\sqrt{-36} =$
$\sqrt{36(-1)}$
$\sqrt{36}\sqrt{-1}$
$6i$

$$cis\theta cis\phi = cis(\theta + \phi)$$
$$\frac{cis\theta}{cis\phi} = cis(\theta - \phi)$$
$$cis(\theta + 2\pi k) = cis\theta \; ; k \in Z$$

$|z^*| = |z|$
$|z|^2 = zz^*$
$|z_1 z_2| = |z_1||z_2|$
$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
$|z_1 z_2 \dots z_n| = |z_1||z_2| \dots |z_n|$
$|z^n| = |z|^n \; n \in Z^+$

$\boxed{|z_1||w| = |zw|}$

$$z^n = r^n cisn\theta = \left(re^{i\theta}\right)^n$$

$i^m = i^{m \bmod 4}$ $\quad m \bmod 4 = remainder \; of \; \frac{m}{4}$

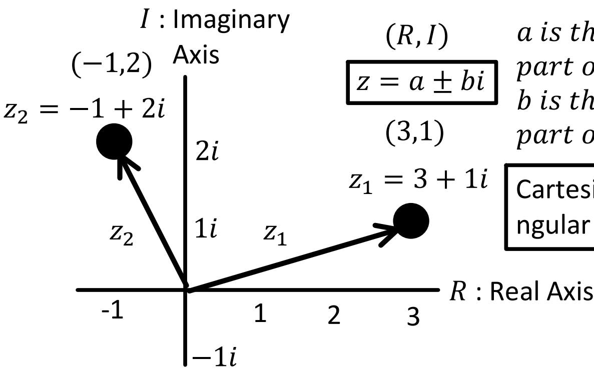
$i^8 = i^0$ $\quad \frac{8}{4} = 2\, r0$
$i^{14} = i^2$ $\quad \frac{14}{4} = 3\, r2$
$i^{29} = i^1$ $\quad \frac{29}{4} = 7\, r1$

$i^{4n+3}, n \in I$
$i^3 i^{4n}$
$-i(i^0)$
$-1(1)$
-1

$4 \bmod 4 = 0$
$8 \bmod 4 = 0$
$14 \bmod 4 = 2$
$29 \bmod 4 = 1$

$z = a + bi \qquad z = r(cos\theta + isin\theta) = re^{i\theta} = r^n cisn\theta$

LA - 3.2 - Graphs/Operations Complex Numbers

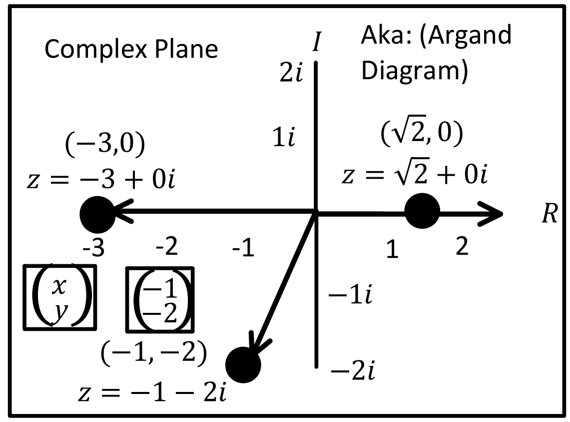


(R, I)

$z = a \pm bi$

a is the real part of z.
b is the imaginary part of z.

Cartesian/Rectangular Form



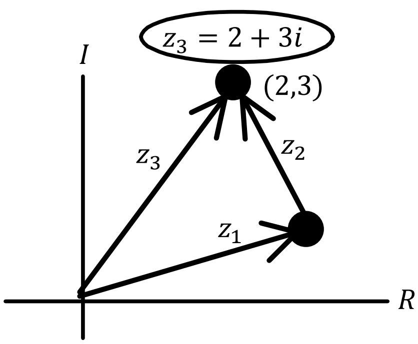
Adding/Subtraction/Distribution

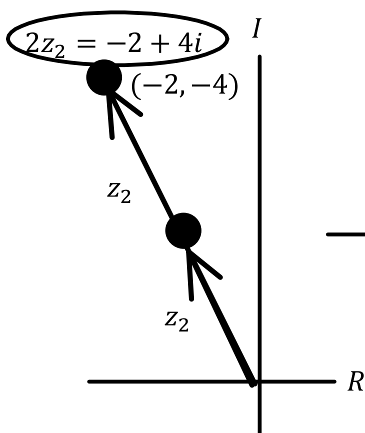
$z_1 = 3 + i$
$z_2 = -1 + 2i$

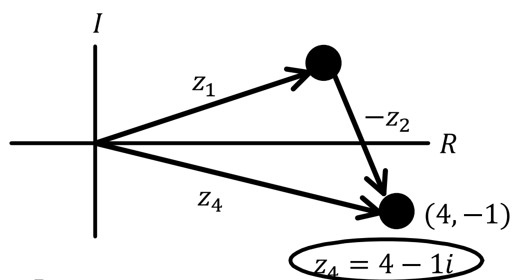
$z_3 = z_1 + z_2$
$z_3 = (3 + i) + (-1 + 2i)$
$z_3 = 2 + 3i$

$2z_2 = 2(-1 + 2i)$
$2z_2 = -2 + 4i$

$z_4 = z_1 - z_2$
$z_4 = 3 + i - (-1 + 2i)$
$z_4 = 4 - 1i$

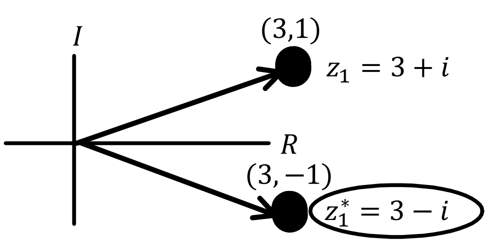


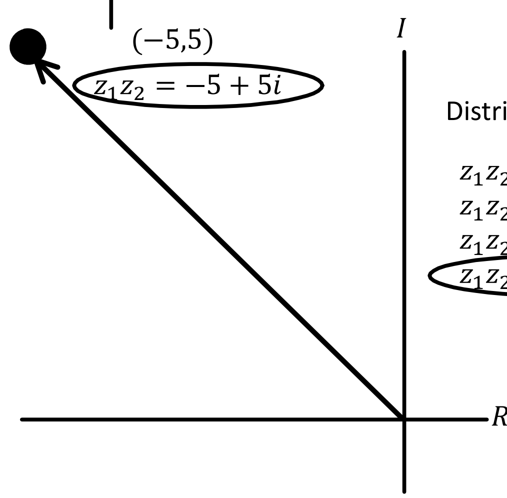




Conjugate

z^* is the conjugate of z
Reflection in x axis





Distribution

$z_1 z_2 = (3 + i)(-1 + 2i)$
$z_1 z_2 = -3 + 5i + 2i^2$
$z_1 z_2 = -3 + 5i + 2(-1)$
$z_1 z_2 = -5 + 5i$

Conjugate

$\frac{10}{2 - i}$

$\frac{10}{2 - i} \times \frac{2 + i}{2 - i}$

$\frac{20 + 10i}{5}$

$4 + 2i$

$(2 - i)(2 + i)$
$4 + 2i - 2i - i^2$
$4 - i^2$
$4 - (-1)$
$4 + 1 = 5$

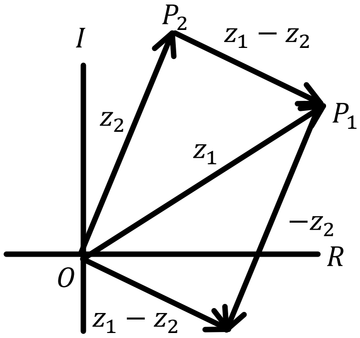
Square Roots

$\sqrt{5 + 12i}$
$\sqrt{(3 + 2i)^2}$
$3 + 2i$

$5 + 12i =$
$5 + 12i + 4 - 4$
$5 + 12i + 4 + 4i^2$
$9 + 12i + 4i^2$
$(3 + 2i)(3 + 2i)$ Factor
$(3 + 2i)^2$

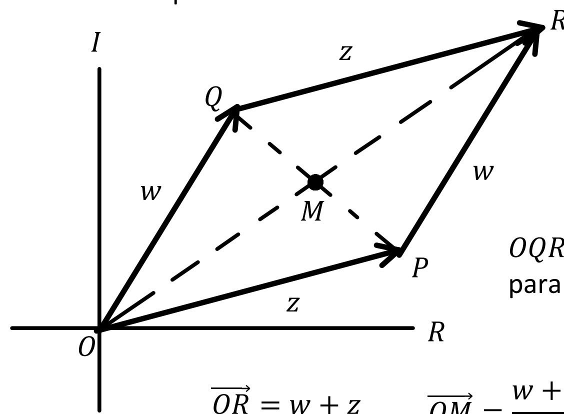
LA - 3.3 - Distance/Midpoint Complex Numbers

Distance



$$\overrightarrow{P_2P_1} = \overrightarrow{P_2O} + \overrightarrow{OP_1}$$
$$\overrightarrow{P_2P_1} = -z_2 + z_1$$
$$\overrightarrow{P_2P_1} = z_1 - z_2$$
$$\left|\overrightarrow{P_2P_1}\right| = |z_1 - z_2|$$

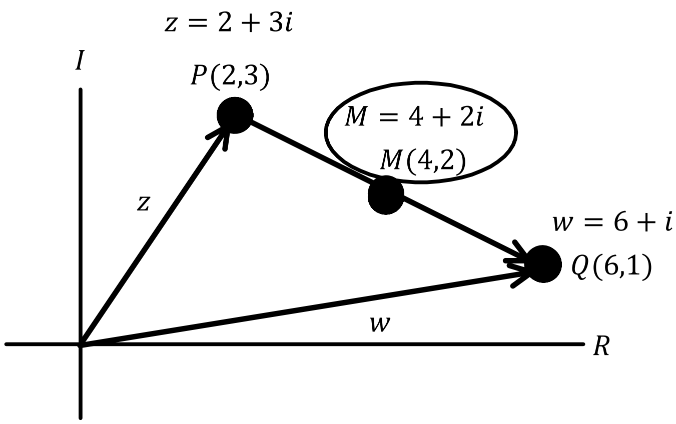
M : Midpoint



$OQRP$ is a parallelgram

$$\overrightarrow{OR} = w + z \qquad \overrightarrow{OM} = \frac{w + z}{2}$$

Distance/Midpoint

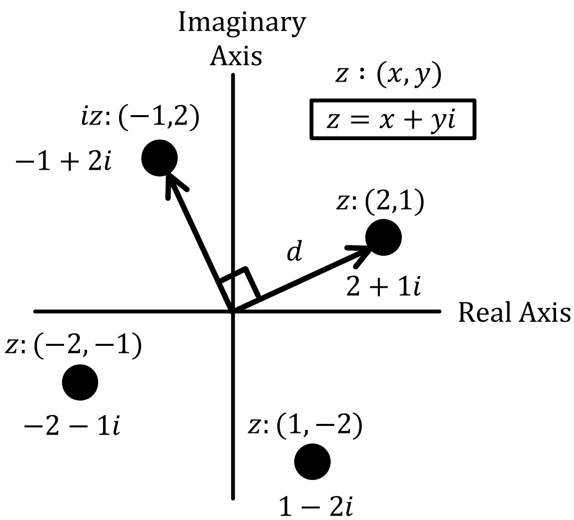


$$w - z = 6 + i - (2 + 3i)$$
$$w - z = 4 - 2i$$

$$|w - z| = \sqrt{4^2 + (-2)^2}$$
$$|w - z| = \sqrt{20} = 2\sqrt{5}$$

Midpoint $PQ = \frac{w+z}{2}$
$$= \frac{6 + i + 2 + 3i}{2}$$
$$= \frac{8 + 4i}{2}$$
Midpoint $PQ = 4 + 2i$

LA - 3.4 - Rotations iz Complex Numbers



z
iz : 90° Rotation
i^2z : 180^o Rotation (From z)
i^2z : 180^o Rotation (From z)
i^3z : 270^o Rotation (From z)
i^4z : z

$z = 2 + i$
$iz = i(2 + 1i)$
$iz = 2i + i^2$
$iz = 2i + (-1)$
$iz = -1 + 2i$

$z = 2 + i$ OR
$i^2z = i^2(2 + 1i)$
$i^2z = -1(2 + 1i)$
$i^2z = -2 - 1i$

$iz = -1 + 2i$
$iiz = i(-1 + 2i)$
$iiz = -1i + 2i^2$
$iiz = -1i + 2(-1)$
$iiz = -2 - 1i$

$z = 2 + i$
$i^3z = i^3(2 + 1i)$
$i^3z = -i(2 + 1i)$
$i^3z = -2i + 1i^2$
$i^3z = -2i + 1(-1)$
$i^3z = -1 - 2i$

$z = 2 + i$
$i^4z = i^4(2 + 1i)$
$i^4z = 1(2 + 1i)$
$i^4z = 2 + 1i$

LA - 3.5 - Polar Complex Numbers

Calculator

MODE	a+bi		i	2nd	.
	re^iθ				

$r : length\ of\ vector : (Modulus)$
$\theta : angle\ (argument: \arg z)$

Domain : $\pi < \theta \le \pi$

$|z| = \sqrt{a^2 + b^2}$

$tan\theta = \frac{b}{a}$

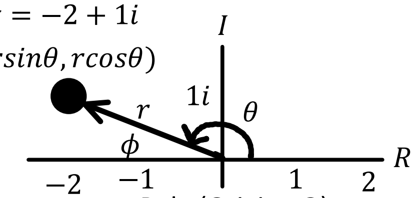
$|z| = \sqrt{(-2)^2 + 1^2}$
$|z| = \sqrt{5}$

$tan\phi = \frac{1}{-2}$

$\phi = \tan^{-1}\left(\frac{1}{2}\right)$

$\phi = 0.46$

$(-2,1)$
$z = -2 + 1i$
$(rsin\theta, rcos\theta)$



Pole (Origin : O)

$cos\theta = \frac{a}{r}$
$a = rcos\theta$

$sin\theta = \frac{b}{r}$
$b = rsin\theta$

$\theta = \pi - 0.46$
$\theta = 2.68$

(r, θ)
$(\sqrt{5}, 2.68)$

Polar Coordinates

$z = a + bi$
$z = rcos\theta + irsin\theta$
$z = r(cos\theta + isin\theta)$

$z = |z|cis\theta$ Polar Form

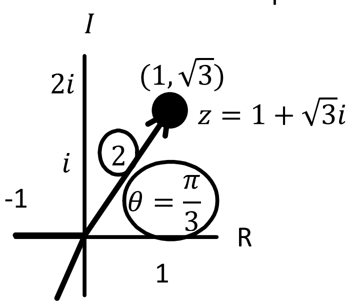
$r = |z| \quad cis\theta = cos\theta + isin\theta = e^{i\theta}$

$e^{i\pi} = -1$

$e : Eulers\ \# = 2.71 ...$

$z = \sqrt{5}cis2.68 = \sqrt{5}e^{2.68i}$ Exponential Form

Convert to Polar & Exponential Form | MATH | CPX | Rect |

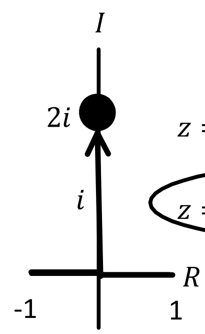


$|r| = \sqrt{(1)^2 + (\sqrt{3})^2}$
$|r| = \sqrt{4}$
$|r| = 2$

$tan\theta = \frac{\sqrt{3}}{1}$

$\theta = \frac{\pi}{3}$

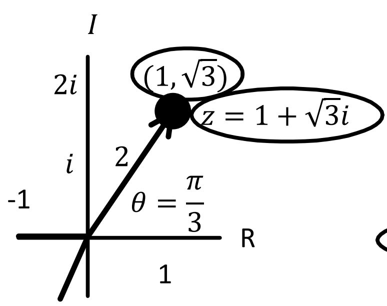
$z = 2cis\left(\frac{\pi}{3}\right) = 2e^{\frac{\pi}{3}i}$



$z = 2i$

$z = 2cis(90^o) = 2e^{\frac{\pi}{2}i}$

Convert to Rectangular Form | MATH | CPX | Polar |

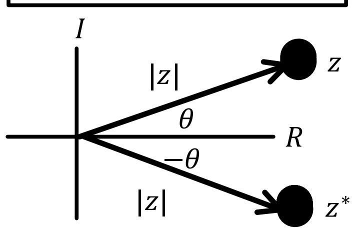


$z = 2cis\frac{\pi}{3}$
$z = 2(\cos\frac{\pi}{3} + isin\frac{\pi}{3})$
$z = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$
$z = 1 + \sqrt{3}i$

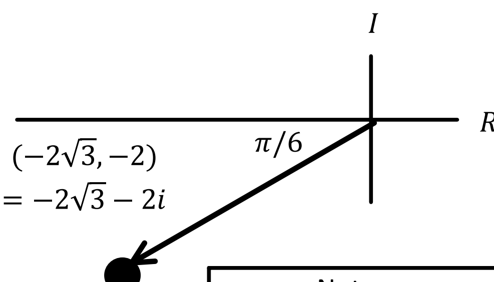
$z = 4cis\left(\frac{-5\pi}{6}\right)$
$z = 4\left(\cos\left(\frac{-5\pi}{6}\right) + isin\left(\frac{-5\pi}{6}\right)\right)$
$z = 4\left(\frac{-\sqrt{3}}{2} + i\left(\frac{-1}{2}\right)\right)$
$z = -2\sqrt{3} - 2i$

Conjugate

z^* *is the conjugate of* z
Reflection in x *axis*



$z^* = |z|cis(-\theta)$



$(-2\sqrt{3}, -2)$
$z = -2\sqrt{3} - 2i$

Note :
$(-1, 90^o) = (1, 270^o)$