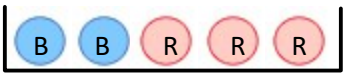
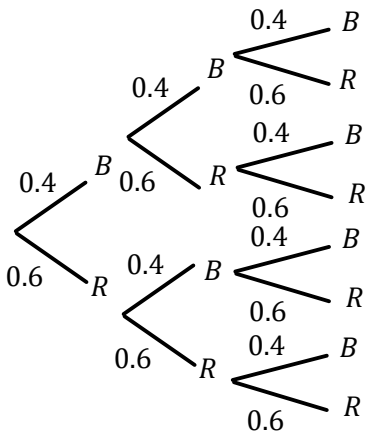
P12 - 2.7 - Binomial Distribution Notes



3 Red/2 Black Balls w/Replacement

$P(R) = \frac{3}{5} = 0.6 = p$ $\quad P(B) = \frac{2}{5} = 0.4 = q$

Binomial Dist!
-Success/Fail
-Constant Prob!
-Independent
-Fixed # of Trials
-Order Doesn't Matter
$P^*(B,B,G) \neq Bin!$



$p(BBB) = 0.4 \times 0.4 \times 0.4 = 0.064 \rightarrow {}_3C_0 = 1$

$p(BBR) = 0.4 \times 0.4 \times 0.6 = 0.096$

$p(BRB) = 0.4 \times 0.6 \times 0.4 = 0.096 \rightarrow {}_3C_1 = 3$

$p(BRR) = 0.4 \times 0.6 \times 0.6 = 0.144$

$p(RBB) = 0.6 \times 0.4 \times 0.4 = 0.096 \rightarrow {}_3C_2 = 3$

$p(RBR) = 0.6 \times 0.4 \times 0.6 = 0.144$

$p(RRB) = 0.6 \times 0.6 \times 0.4 = 0.144$

$p(RRR) = 0.6 \times 0.6 \times 0.6 = 0.216 \rightarrow {}_3C_3 = 1$

100%

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \end{array}$$

Binomial Distribution Order Doesn't Matter

$p(x) = {}_nC_x p^x q^{n-x}$

$n = \#\ of\ Trials$
$x = \#\ of\ Successes$

Note $p(0B) = p(3R)$

$p(0B) = {}_3C_0 (0.4)^0 (0.6)^{3-0}$
$p(0B) = 1(1)(0.216)$
$p(0B) = 0.216$

$p(3B) = {}_3C_3 (0.4)^3 (0.6)^{3-3}$
$p(3B) = 1(0.064)(1)$
$p(3B) = 0.064$

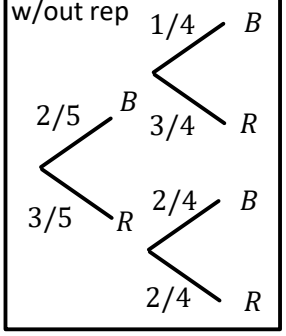
2nd DISTR binompdf @A

$binomialpdf(3,0.4,3) = 0.064$

$p(1B) = {}_3C_1 (0.4)^1 (0.6)^{3-1}$
$p(1B) = 3(0.4)(0.6)^2$
$p(1B) = 0.432$

$p(1B) = p(RRB) + p(RBR) + p(BRR)$
$p(1B) = 0.144 + 0.144 + 0.144$
$p(1B) = 0.432$

without replacement
w/out rep



$p(\geq 1B) = p(1B) + p(2B) + p(3B)$
$p(\geq 1B) = 0.432 + 0.288 + 0.064$
$p(\geq 1B) = 0.784$

$p(\geq 1B) = 1 - p(0B)$ All-None
$p(\geq 1B) = 1 - 0.216$
$p(\geq 1B) = 0.784$

$p(\leq 1B) = p(0B) + p(1B)$
$p(\leq 1B) = 0.216 + 0.432$
$p(\leq 1B) = 0.648$

2nd DISTR binomicdf @B

$binomialcdf(3,0.4,1) = 0.648$

3 Red/2 Black Balls wout/Replacement

Note: $p(2B,1R) = p(B,B,R) \times {}_3C_2 = \frac{1}{10} \times 3 = \frac{3}{10}$

Order Matters

$p(B,B,R) = \frac{2}{5} \times \frac{1}{4} \times \frac{3}{3} = \frac{6}{60} = \frac{1}{10}$

${}_2P_2 \times {}_3P_1 = 6 \quad {}_5P_3 = 60$

Order Doesn't Matter

$p(2B,1R) = p(B,B,R) + p(B,R,B) + p(R,B,B)$
$p(2B,1R) = \frac{6}{60} + \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} = \frac{18}{60} = \frac{3}{10}$

${}_2C_2 \times {}_3C_1 = 3 \quad {}_5C_3 = 10$

$10B's, 10R's$
w/replacement
Order matters

w/replacement ≠
Order doesn't matter

$p(5B,5R) = {}_{10}C_5 \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{10} = 0.246$

${}_{10}C_5 = 252 \quad \frac{252 \times 10^{10}}{20^{10}} = 0.246$

w/out replacement
Order matters

$p(5B,5R) = \frac{{}_{10}P_5 \times {}_{10}P_5 \times {}_{10}C_5}{{}_{20}P_{10}} = 0.3437$

w/out replacement
Order doesn't matter

$\frac{{}_{10}C_5 \times {}_{10}C_5}{{}_{20}C_{10}} = 0.3437$