P12 - 2.9 - Coins Probability

Choose 3, one at a time
Order matters

w/out rep Coins. let G=Gold (6), let S=Silver (4), let B=Bronze (3)

Case #3 :
All different

GSB	SGB	BGS
GBS	SBG	BSG

$3! = 6$

$$p(1G,1S,1B) = p(G,S,B) + p(G,B,S) + p(S,B,G) + p(S,G,B) + p(B,G,S) + p(B,S,G)$$

$$p(1G,1S,1B) = \frac{6}{13}\times\frac{4}{12}\times\frac{3}{11}+\frac{6}{13}\times\frac{3}{12}\times\frac{4}{11}\ldots\times\frac{3}{13}\times\frac{4}{12}\times\frac{6}{11}=\frac{432}{1716}=\frac{72}{286}=\frac{36}{143}$$

$${}_6C_1 \times {}_4C_1 \times {}_3C_1 \times 3! = 6\times 4\times 3\times 3! = 72\times 6 = 432 \qquad {}_{13}P_3 = 1716$$

C#1+C#2+C#3

$3 + 18 + 6 = 27 = 3!$

$$p(all\ same) = \frac{6}{13}\times\frac{5}{12}\times\frac{4}{11}\times\cdots\times\frac{3}{13}\times\frac{2}{12}\times\frac{1}{11}=\frac{150}{1716}=\frac{25}{286}$$

GGG
SSS
BBB

Case #1 :
All Same

3

$$432 + 150 + 1134 = 1716$$

$$p(2\ same) = \frac{6}{13}\times\frac{5}{12}\times\frac{4}{11}+\cdots\frac{3}{13}\times\frac{2}{12}\times\frac{4}{11}=\frac{1134}{1716}=\frac{189}{286}$$

$$p(2G) = \frac{6}{13}\times\frac{5}{12}\times\frac{4}{11}+\cdots\frac{3}{13}\times\frac{6}{12}\times\frac{5}{11}=\frac{540}{1716}=\frac{105}{286}$$

$$p(2G) = p(2G,1B) + p(2G,1S)$$

$$p(2G,1B) = \frac{6}{13}\times\frac{5}{12}\times\frac{3}{11}+\cdots\frac{3}{13}\times\frac{6}{12}\times\frac{5}{11}=\frac{270}{1716}=\frac{45}{286}$$

GGB	SSG	BBG
GBG	SGS	BGB
BGG	GSS	GBB
GGS	SSB	BBS
GSG	SBS	BSB
SGG	BSS	SBB

$${}_3C_1 \times {}_2C_1 \times \frac{3!}{2!} = 18$$

$${}_3P_2 = 6$$

$${}_3C_2 = 3$$

Case #2 :
2 Same

Choose 3 at one time/one at a time
Order doesn't matter

w/out rep

$$p(1G,1S,1B) = \frac{72}{286} = \frac{36}{143}$$

GSB=GBS=SGB=SBG=BGS=BSG

Order matters=Order doesn't matter

$${}_6C_1 \times {}_4C_1 \times {}_3C_1 = 72 \qquad {}_{13}C_3 = 13\times 12\times 11 = 286$$

$$p(all\ same) = \frac{25}{286}$$

GGG
SSS
BBB

$$72 + 25 + 189 = 286$$

$${}_6C_3 + {}_4C_3 + {}_3C_3 = 25$$

$$p(2\ same) = \frac{189}{286}$$

GGB=GBG=BGG=GGS=GSG=SGG
SSG=SGS=GSS=SSB=SBS=BSS
BBG=BGB=GBB=BBS=BSB=SBB

$${}_6C_2 \times {}_4C_1 + {}_6C_2 \times {}_3C_1 + {}_4C_2 \times {}_6C_1 + {}_4C_2 \times {}_3C_1 + {}_3C_2 \times {}_6C_1 + {}_3C_2 \times {}_4C_1 = 189$$

$$p(2G,1S) + p(2G,1B) + p(2S,1G) + p(2S,1B) + p(2B,1G) + p(2B,1S)$$

$15\times 4 + 15\times 3 = 105$

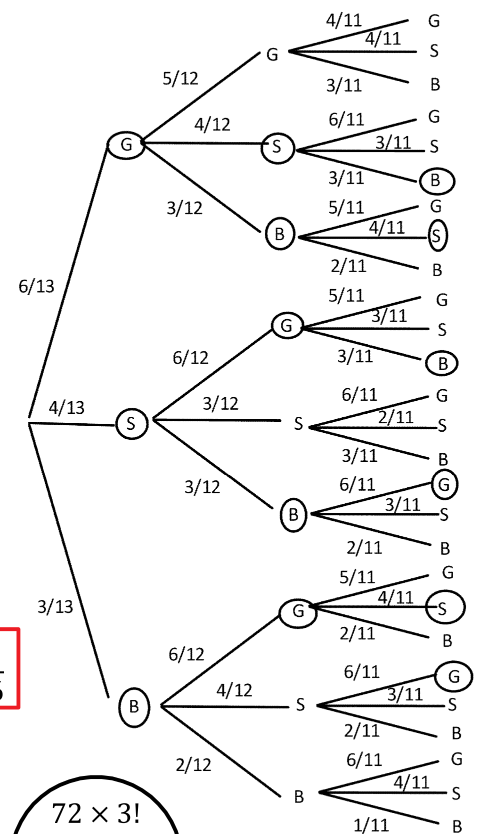
$6\times 6 + 6\times 3 = 54$

$3\times 6 + 3\times 4 = 30$

$105 + 54 + 30 = 189$

Choose 3, one at a time
Order matters/doesn't

w/rep Tree $\frac{6}{13},\frac{4}{13},\frac{3}{13}$



$$p(G,S,B) = \frac{6}{13}\times\frac{4}{13}\times\frac{3}{13} = \frac{72}{2197}$$

$$p(1G,1S,1B) = 3!\times\frac{72}{2197} = \frac{432}{2196}$$

$$p(all\ same) = \left(\frac{6}{13}\right)^3 + \left(\frac{4}{13}\right)^3 + \left(\frac{3}{13}\right)^3 = \frac{307}{2197}$$

$$p(2\ same) = 3\left(\frac{6}{13}\right)^2\times\frac{4}{13} + 3\left(\frac{6}{13}\right)^2\times\frac{3}{13} + \cdots 3\left(\frac{3}{13}\right)^2\times\frac{4}{13} = \frac{1458}{2197}$$

$72\times 3!$
$+ \quad 307$
$+ \quad 1458$
$= 2197$