S12 - 3.13 - Linear Regression/Correlation Notes

Least Squares Method

$n = 7$

x	y	xy	x^2	y^2
1	1.5	1.5	1	2.25
2	3.8	7.6	4	14.44
3	6.7	20.1	9	44.89
4	9.0	36	16	81
5	11.2	56	25	125.44
6	13.6	81.6	36	184.96
<u>7</u>	<u>16</u>	<u>112</u>	<u>49</u>	256
28	*61.8*	*314.8*	140	708.98

$\sum x = 28$ $\quad \sum xy = 314.8$ $\quad \sum y^2 = 708.98$

$\sum y = 61.8$ $\quad \sum x^2 = 140$

$$y = mx + b$$

$$m = \frac{n\sum x - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$m = \frac{7(28) - (28)(61.8)}{7(140) - (28)^2}$$

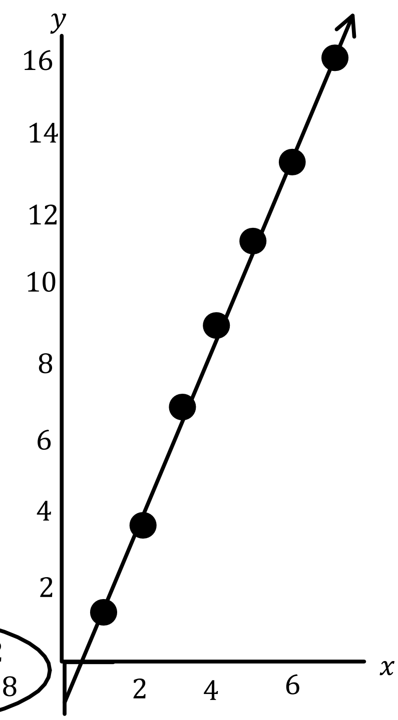
$$m = \frac{473.2}{196} = 2.4142857$$

$$b = \frac{\sum y - m\sum x}{n}$$

$$b = \frac{61.8 - (2.4142857)(28)}{7}$$

$$b = -0.828571$$

$$y = 2.41x - 0.83$$



Correlation Coefficient : $-1 \le r \le 1$

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2)(n\sum y^2 - (\sum y)^2)}}$$

$$r = \frac{7(314.8) - (28)(61.8)}{\sqrt{(7(140) - (28)^2)(7(708.98) - (61.8)^2)}}$$

$$r = \frac{473.2}{\sqrt{(196)1143.62)}}$$

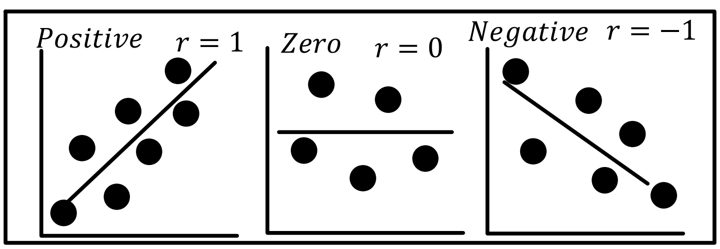
$$r = 0.999$$

OR

$$r = \frac{1}{n-1}\sum\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

$\bar{x} = 4$ $\quad \bar{y} = 8.82$

$s_y = 2.16$ $\quad s_y = 5.218$



$$r = \frac{1}{7-1}\left(\left(\frac{1-4}{2.16}\right)\left(\frac{1.5-8.82}{5.218}\right) + \left(\frac{2-4}{2.16}\right)\left(\frac{3.8-8.82}{5.218}\right) + \left(\frac{3-4}{2.16}\right)\left(\frac{6.7-8.82}{5.218}\right) + \left(\frac{4-4}{2.16}\right)\left(\frac{9.0-8.82}{5.218}\right) + \left(\frac{5-4}{2.16}\right)\left(\frac{11.2-8.82}{5.218}\right) + \left(\frac{6-4}{2.16}\right)\left(\frac{13.6-8.82}{5.218}\right) + \left(\frac{7-4}{2.16}\right)\left(\frac{16-8.82}{5.218}\right)\right)$$

$$r = 1.6(1.948 + 0.891 + 0.188 + 0 + 0.1854 + 0.848 + 1.91)$$

$$r = 0.995$$

$y = mx + b$

$y = a + bx$

$y^* = -0.78 + 2.4x$

$a = \bar{y} - b\bar{x}$

$a = 5.218 - 2.4(4)$

$a = -0.78$

$$b = r\frac{s_y}{s_x}$$

$$b = 0.995\frac{5.218}{2.16}$$

$$b = 2.4$$

$$R^2 = \frac{\sum(y^* - \bar{y})^2}{\sum(y_i - \bar{y})^2} \;;(0,1)$$

$R^2 = 0.98$

$R = 0.99$