S12 - 3.8 - Geometric Distribution Notes

6-sided die. $p(1st\ 6\ on\ 4th\ roll)$

fail, fail, fail, success $p(6) = \frac{1}{6}$ $P(\bar{6}) = \frac{5}{6}$

$$p(X = x) = q^{x-1} \times p$$

$$p(X = 4) = \left(\frac{5}{6}\right)^{4-1} \times \left(\frac{1}{6}\right)$$

$$= \left(\frac{5}{6}\right)^{3} \times \left(\frac{1}{6}\right) = \frac{125}{1296}$$

4% of townspeople are teachers. $p(10th\ person\ is\ a\ teacher)$

$let\ T = Teacher$ $p(T) = 0.04$ $p(\bar{T}) = 1 - 0.04 = 0.96$

$$p(X = 10) = (0.96)^{10-1} \times (0.04)$$
$$= 0.0277$$

$$\mu = \frac{1}{p} = \frac{1}{0.04} = 25$$

$$\sigma^2 = \left(\frac{1}{p}\right)\left(\frac{1}{p} - 1\right)$$
$$\sigma^2 = 25(25 - 1)$$
$$\sigma^2 = 600$$

$$\sigma = \sqrt{\sigma^2}$$
$$\sigma = \sqrt{600}$$
$$\sigma = 24.49$$

2% of tires are defective. Test 100 tires. $p(8th\ tire\ is\ defective)$

$let\ D = Defective$ $p(D) = 0.02$ $P(\bar{D}) = 1 - 0.02 = 0.98$

$$p(X = 8) = (0.98)^{8-1} \times (0.02)$$
$$= 0.01736$$

How many tires would you expect to test until you find the first defective one.

$$\mu = \frac{1}{p} = \frac{1}{0.02} = 50$$

$p(1st\ defective\ tire\ among\ 1st\ 5\ samples)$

$$p(X \le 5) = p(X = 1) + p(X = 2) + p(X = 3) + p(X = 4) + p(X = 5)$$
$$= q^{x-1}p + q^{x-1}p + q^{x-1}p + q^{x-1}p + q^{x-1}p$$
$$= q^{o}p + q^{1}p + q^{2}p + q^{3}p + q^{4}p$$
$$= p(1 + q^1 + q^2 + q^3 + q^4)$$
$$= 0.02(1 + 0.98^1 + 0.98^2 + 0.98^3 + 0.98^4)$$
$$= 0.096$$

$$p(X \le x) = 1 - q^x$$
$$p(X \le 5) = 1 - 0.98^5$$
$$= 0.096$$

$$p(X < 6) = p(X \le 5)$$
$$= 0.096$$

$$p(X \le x) + p(X > x) = 1$$
$$p(X > 5) = 1 - p(X \le 5)$$
$$= 1 - 0.096$$
$$= 0.904$$

$$p(X > x) = q^x$$
$$p(X > 5) = q^x$$
$$= 0.98^5$$
$$= 0.904$$

$$p(5 \le X < 8) = p(5 \le X \le 7)$$
$$= p(X \le 7) - p(x \le 5)$$
$$= 0.132 - 0.096$$
$$= 0.036$$

$$p(X \le 7) = 1 - q^x$$
$$= 1 - 0.98^7$$
$$= 0.132$$